Moments and Other Measures in Terms of Expectations

Data Science and A.I. Lecture Series

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Moments and Other Measures in Terms of Expectations

Moments:

- The r^{th} order moment about any point A of a variable X is given by:

$$\mu'_r = \sum_{i=1}^{n} p_i (x_i - A)^r \tag{1}$$

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- This is derived by replacing frequencies with probabilities.
- If X is a continuous random variable with probability density function $f(x)$, then:

$$\mu'_r = \int_{-\infty}^{\infty} (x - A)^r f(x) dx \tag{2}$$

Central Moments

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- The expectation form:

$$\mu_r = E[(X - \mu)^r] \tag{5}$$

Variance

Definition:

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- Using moments about the origin:

$$V(X) = \mu_2' - (\mu_1')^2 \tag{7}$$

  where μ_1' and μ_2' are the first and second moments about the origin.

Theorem: Variance Scaling Property

Statement:

- If X is a random variable and a, b are constants, then:

$$V(aX + b) = a^2 V(X) \tag{8}$$

Proof of Theorem

- By definition of variance:

$$V(aX + b) = E[(aX + b - E[aX + b])^2] \tag{9}$$

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- Simplifying:

$$= E[(aX-aE[X])^2] = E[a^2(X-E[X])^2] \tag{11}$$

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- Using expectation properties:

$$= a^2E[(X-E[X])^2] = a^2V(X) \tag{12}$$

Example 7: Variance Calculation

Given Probability Distribution Calculate Variance:

X	$p(X)$
-2	0.15
-1	0.30
0	0
1	0.30
2	0.25

(i) Computing $V(X)$:

- Variance formula:

$$V(X) = E[X^2] - (E[X])^2 \tag{13}$$

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(i) Computing $V(X)$:

- Variance formula:

$$V(X) = E[X^2] - (E[X])^2 \tag{13}$$

- Using values from Example 6:

$$V(X) = 2.2 - (0.2)^2 = 2.2 - 0.04 = 2.16 \tag{14}$$

Variance of a Linear Transformation

(ii) Computing $V(2X + 3)$:

- Using the theorem: $V(aX + b) = a^2 V(X)$

$$V(2X + 3) = 4V(X) \tag{15}$$

Variance of a Linear Transformation

(ii) Computing $V(2X + 3)$:

- Using the theorem: $V(aX + b) = a^2 V(X)$

$$V(2X + 3) = 4V(X) \tag{15}$$

- Substituting $V(X) = 2.16$:

$$V(2X + 3) = 4(2.16) = 8.64 \tag{16}$$

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