# Binomial Distribution

## Data Science and A.I. Lecture Series

Bindeshwar Singh Kushwaha

PostNetwork Academy

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# Binomial Probability Function

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$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \tag{1}$$

where:

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- $p$ = probability of success in each trial
- $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ is the binomial coefficient

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- The term $(1-p)^{n-k}$ accounts for the probability of the remaining $(n-k)$ failures.
- Multiplying these terms together gives the probability of observing exactly $k$ successes in $n$ trials.

# Example Calculation

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- Compute probability:
$$P(X = 2) = 3 \times (0.5)^2 \times (0.5)^1 \tag{3}$$
- Final result:
$$P(X = 2) = 3 \times 0.25 \times 0.5 = 0.375 \tag{4}$$

# Binomial Distribution

A discrete random variable $X$ is said to follow a binomial distribution with parameters $n$ and $p$ if it assumes only a finite number of non-negative integer values and its probability mass function is given by:

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- If $X$ is a binomially distributed random variable with parameters $n$ and $p$, then we may write it as $X \sim B(n, p)$.
- If $X$ and $Y$ are two binomially distributed independent random variables with parameters $(n_1, p)$ and $(n_2, p)$ respectively, then their sum also follows a binomial distribution with parameters $n_1 + n_2$ and $p$. However, if the probability of success is not the same for the two random variables, this property does not hold.

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- More than 6 heads

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# Solution

Given:

- Number of trials: $n = 6$
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- Probability of failure: $q = 1 - p = \frac{1}{2}$
- Let $X$ be the number of successes in $n$ trials.
- By binomial distribution:

$$P(X = x) = \binom{6}{x} p^x q^{6-x}$$

# Calculations

- **(i) Exactly 3 heads:**

$$P(X = 3) = \binom{6}{3}\left(\frac{1}{2}\right)^6 = \frac{20}{64} = \frac{5}{16}$$

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$$P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= \binom{6}{0} \left(\frac{1}{2}\right)^6 + \binom{6}{1} \left(\frac{1}{2}\right)^6 + \binom{6}{2} \left(\frac{1}{2}\right)^6$$

$$= \frac{1}{64} + \frac{6}{64} + \frac{15}{64} = \frac{22}{64} = \frac{11}{32}$$

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- **(iii) More than 3 heads:**

$$P(X > 3) = P(X = 4) + P(X = 5) + P(X = 6)$$

$$= \frac{15}{64} + \frac{6}{64} + \frac{1}{64} = \frac{22}{64} = \frac{11}{32}$$

# More Calculations

- **(iv) At most 3 heads:**

$$P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$

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- **(vi) More than 6 heads:**

$$P(X > 6) = 0$$

(impossible event)

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