Addition, Multiplication Theorem of Expectation and Covariance

Data Science and A.I. Lecture Series

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Outline

- Introduction

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- Addition Theorem of Expectation

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- Proof of Addition Theorem

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- Multiplication Theorem of Expectation

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- It provides a measure of the central tendency of a random variable.
- We discuss two key theorems: Addition and Multiplication Theorems of Expectation.

Addition Theorem of Expectation

Theorem

If X and Y are two random variables, then:

$$E(X + Y) = E(X) + E(Y)$$

- The expected value of the sum of two random variables is equal to the sum of their expected values.

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- This holds for any finite number of random variables:

$$E(X_1 + X_2 + \cdots + X_n) = E(X_1) + E(X_2) + \cdots + E(X_n)$$

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- The theorem holds regardless of whether the variables are independent or dependent.

Proof of Addition Theorem

- By definition, expectation is computed as:

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- Separating terms,

$$E(X + Y) = \sum_{x} xP(X = x) + \sum_{y} yP(Y = y) = E(X) + E(Y)$$

Multiplication Theorem of Expectation

Theorem

If X and Y are two independent random variables, then:

$$E(XY) = E(X)E(Y)$$

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- The expected value of the product of two independent random variables is the product of their expected values.
- This does not necessarily hold if X and Y are dependent.

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- Separating sums,

$$E(XY) = \left(\sum_{x} xP(X = x)\right)\left(\sum_{y} yP(Y = y)\right) = E(X)E(Y)$$

Covariance

- For a bivariate frequency distribution, covariance between two variables X and Y is defined as:

$$\text{Cov}(X, Y) = \frac{\sum f_i(x_i - \bar{X})(y_i - \bar{Y})}{\sum f_i} \tag{1}$$

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- For a bivariate probability distribution:

$$\mathrm{Cov}(X, Y) = \begin{cases} \sum (x_i - \mathbb{E}[X])(y_i - \mathbb{E}[Y]) p_{ij}, & \text{discrete case} \\ \int \int (x - \mathbb{E}[X])(y - \mathbb{E}[Y]) f(x, y) dxdy, & \text{continuous case} \end{cases} \tag{2}$$

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- Using expectation:

$$\mathrm{Cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y] \tag{3}$$

- If X and Y are independent, then $\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$
  Hence, $\mathrm{Cov}(X, Y) = 0$

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Thank You!