# Cumulative Distribution Function (CDF)

## Data Science and A.I. Lecture Series

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# Definition: Discrete Frequency Distribution

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- The CDF is always non-decreasing and satisfies:

$$0 \leq F(x) \leq 1$$

# Example 7: Discrete CDF

- A random variable $X$ has the following probability function:

| $X$ | $p(x)$ | $F(x) = P[X \leq x]$ |
|---|---|---|
| 0 | 0 | 0 |
| 1 | $\frac{1}{10}$ | $\frac{1}{10}$ |
| 2 | $\frac{1}{5}$ | $\frac{3}{10}$ |
| 3 | $\frac{1}{5}$ | $\frac{5}{10}$ |
| 4 | $\frac{3}{10}$ | $\frac{8}{10}$ |
| 5 | $\frac{1}{100}$ | 0.81 |
| 6 | $\frac{1}{50}$ | 0.83 |
| 7 | $\frac{17}{100}$ | 1 |

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- The function $F(x)$ is non-decreasing and satisfies:

$$\lim_{x \to -\infty} F(x) = 0, \quad \lim_{x \to \infty} F(x) = 1$$

# Example 8: Continuous CDF

- The diameter $X$ of a cable is a continuous random variable with probability density function (p.d.f.):

$$f(x) = \begin{cases} 6x(1-x), & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

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- Find the cumulative distribution function $F(x)$.

# Solution: Continuous CDF

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- Final result:

$$F(x) = \begin{cases} 0, & x < 0 \\ 3x^2 - 2x^3, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

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