Bivariate Discrete Random Variables

Data Science and A.I. Lecture Series

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- Example: Consider placing three balls b_1, b_2, b_3 randomly in three cells.
- The number of balls in a cell and the number of occupied cells form discrete random variables.

Table 6.1: Possible Outcomes of Placing Three Balls in Three Cells

X (Number of balls in Cell 1) can take values: $0, 1, 2, 3$
Y (Number of occupied cells) can take values: $1, 2, 3$
The possible ordered pairs (X, Y) are:

$$(0, 1), (0, 2), (0, 3), (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)$$

Arrangement	Cell 1	Cell 2	Cell 3
1	b_1	b_2	b_3
2	b_1	b_3	b_2
3	b_2	b_1	b_3
4	b_2	b_3	b_1
5	b_3	b_1	b_2
6	b_3	b_2	b_1
7	b_1, b_2	b_3	–
8	b_1, b_2	–	b_3
9	–	b_1, b_2	b_3
10	b_1, b_3	b_2	–
11	b_1, b_3	–	b_2
12	–	b_1, b_3	b_2
13	b_2, b_3	b_1	–
14	b_2, b_3	–	b_1
15	–	b_2, b_3	b_1
16	b_1	b_2, b_3	–
17	b_1	–	b_2, b_3
18	–	b_1	b_2, b_3
19	b_2	b_3, b_1	–
20	b_2	–	b_3, b_1
21	–	b_2	b_3, b_1
22	b_3	b_1, b_2	–
23	b_3	–	b_1, b_2
24	–	b_3	b_1, b_2
25	b_1, b_2, b_3	–	–
26	–	b_1, b_2, b_3	–
27	–	–	b_1, b_2, b_3

Joint Probability Mass Function

The joint probability mass function (PMF) $p(x, y)$ is defined as:

$$p(x, y) = P(X = x, Y = y)$$

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Joint Probability Table:

$X \backslash Y$	1	2	3	$P(X)$
0	$\frac{2}{27}$	$\frac{6}{27}$	0	$\frac{8}{27}$
1	0	$\frac{6}{27}$	$\frac{6}{27}$	$\frac{12}{27}$
2	0	$\frac{6}{27}$	0	$\frac{6}{27}$
3	$\frac{1}{27}$	0	0	$\frac{1}{27}$
$P(Y)$	$\frac{3}{27}$	$\frac{18}{27}$	$\frac{6}{27}$	1

Marginal Probability Distributions

Marginal PMF of X:

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Marginal PMF of Y:

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Conditional Probability Mass Function

The conditional probability mass function is given by:

$$P(X = x \mid Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}$$

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Example: Conditional PMF of X given Y=2

$$P(X = 0 \mid Y = 2) = \frac{6}{18} = \frac{1}{3}, \quad P(X = 1 \mid Y = 2) = \frac{6}{18} = \frac{1}{3}, \quad P(X = 2 \mid Y = 2) = \frac{6}{18} = \frac{1}{3}$$

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