Vectors in $\mathbb{R}^n$ and $\mathbb{C}^n$

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PostNetwork Academy

Outline of Presentation

- Introduction to Vectors

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- Visualization of Vectors in $\mathbb{R}^3$

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- Vector Operations with Examples
- Properties of Vectors
- Dot (Inner) Product
- Orthogonal Vectors

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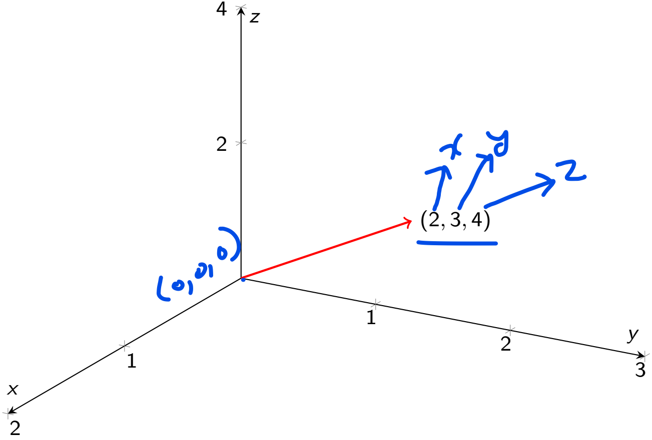
- In mathematics, vectors are elements of vector spaces and can exist in different dimensions.
- Common vector spaces include real number space ($\mathbb{R}^n$) and complex number space ($\mathbb{C}^n$).

Example - 3D Vectors Visualization



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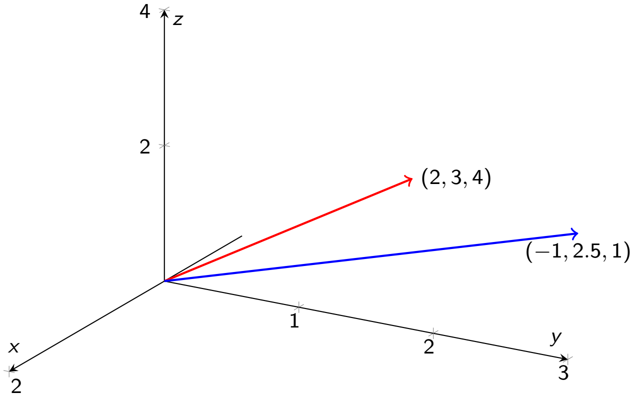

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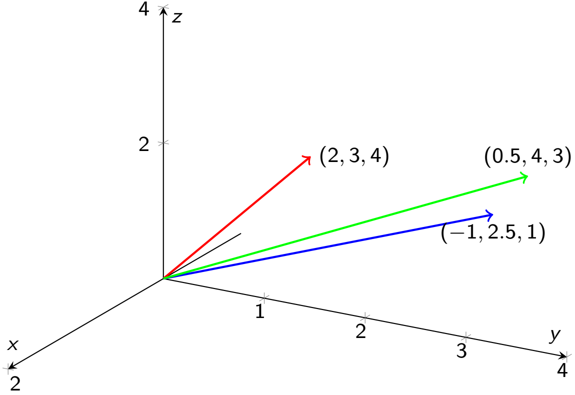

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scalar ∈ R

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- If $u = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, $v = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$, then:
$$2u - 3v = \begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix} + \begin{bmatrix} -9 \\ -3 \\ -6 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 2 \end{bmatrix}$$

Theorem - Properties of Vectors with Examples

- $(u + v) + w = u + (v + w)$
  Example: Let $u = (1, 2)$, $v = (3, 4)$, $w = (5, 6)$, then:
  $$((1, 2) + (3, 4)) + (5, 6) = (4, 6) + (5, 6) = (9, 12)$$
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- If $u \cdot v = 0$, then u and v are orthogonal (perpendicular).

Example 1.3 - Dot Product

- Let $u = (1, 2, 3)$, $v = (4, 5, -1)$, and $w = (2, 7, 4)$.

$$u \cdot v = (1)(4) + (2)(5) + (3)(-1) = 4 + 10 - 3 = 9$$

$$u \cdot w = (1)(2) + (2)(7) + (3)(4) = 2 + 14 + 12 = 28$$

$$v \cdot w = (4)(2) + (5)(7) + (-1)(4) = 8 + 35 - 4 = 39$$

Scalars

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