Differential Equations

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Question

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What are differential equations, their types, and how can they be solved using the method of separation of variables? What are some examples of each type?

Definition of Differential Equations

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This equation shows that the rate of change of y with respect to x is equal to $3x^2$.

Types of Differential Equations with Examples

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- Homogeneous vs. Non-Homogeneous Equations – based on the presence of a free term. Example of Homogeneous:

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$

  Example of Non-Homogeneous:

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x$$

Theory: Separation of Variables

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- Integrate both sides separately.
- Solve for y, if possible.

Example 1: Basic Separable Equation

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Solution: Rewrite as:

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Integrating both sides:

$$\int dy = \int 3x^2 dx$$

$$y = x^3 + C$$

Example 2: Homogeneous Equation

Solve:

$$x\frac{dy}{dx} = y$$

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Example 2: Homogeneous Equation

Solve:

$$x\frac{dy}{dx} = y$$

Solution: Rewrite as:

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$$\frac{dy}{y} = \frac{dx}{x}$$

Integrating both sides:

$$\ln y = \ln x + C$$

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Separating variables:

$$\frac{dy}{y} = \frac{dx}{x}$$

Integrating both sides:

$$\ln y = \ln x + C$$

Taking exponent:

$$y = Cx$$

Example 3: Exponential Decay

Solve:

$$\frac{dy}{dx} = -ky$$

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Integrating:

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Solution: Separating variables:

$$\frac{dy}{y} = -kdx$$

Integrating:

$$\ln y = -kx + C$$

Taking exponent:

$$y = Ce^{-kx}$$

Example 4: Population Growth

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$$\frac{dP}{dt} = kP$$

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Solution: Separating variables:

$$\frac{dP}{P} = kdt$$

Integrating:

$$\ln P = kt + C$$

Taking exponent:

$$P = Ce^{kt}$$

Example 5: Newton's Law of Cooling

Newton's Law of Cooling states:

$$\frac{dT}{dt} = -k(T - T_a)$$

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Integrating:

$$\ln|T - T_a| = -kt + C$$

Example 5: Newton's Law of Cooling

Newton's Law of Cooling states:

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Solution: Separating variables:

$$\frac{dT}{T - T_a} = -kdt$$

Integrating:

$$\ln|T - T_a| = -kt + C$$

Taking exponent:

$$T = T_a + Ce^{-kt}$$

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- The method of separation of variables.

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In this presentation, we covered:

- The definition of differential equations.
- Different types of differential equations with examples.
- The method of separation of variables.
- Five solved examples using this method.