Bivariate Discrete Cumulative Distribution Function

Data Science and A.I. Lecture Series

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PostNetwork Academy

Joint and Marginal Distribution Functions for Discrete Random Variables

- **Two-Dimensional Joint Distribution Function**
  - The distribution function of the two-dimensional random variable (X, Y) for all real x and y is defined as:

$$F(x, y) = P(X \leq x, Y \leq y)$$

Joint Probability Table:

$X \backslash Y$	1	2
1	0.1	0.2
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Joint Probability Table:

$X \backslash Y$	0	1
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