Bivariate Continuous Random Variables

Data Science and A.I. Lecture Series

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- Conditional distributions and stochastic independence will also be explored.
- Problems and examples will reinforce the concepts.

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- Understand joint and marginal distributions.
- Derive conditional distributions and density functions.
- Analyze stochastic independence of two continuous random variables.
- Solve problems involving two-dimensional continuous random variables.

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- Examples:
  - The position (X, Y) of a particle moving in a plane.
  - Temperature and humidity recorded at a location.

Joint and Marginal Distribution and Density Functions

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- The marginal densities are obtained by integrating out the other variable:

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y) \, dy, \quad f_Y(y) = \int_{-\infty}^{\infty} f(x, y) \, dx$$

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- These are useful in prediction and statistical inference.

Stochastic Independence of Two Continuous Random Variables

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- If dependence exists, conditional probabilities are used.

Problems on Two-Dimensional Continuous Random Variables

The joint probability density function of (X, Y) is given by

$$f(x, y) = k(2x + y), \quad 0 < x < 1, \quad 0 < y < 2$$

Find the value of k.

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$$\int_0^1 \int_0^2 k(2x+y)\,dy\,dx = 1$$

$$k\left[\int_0^1 \left(2xy+\frac{y^2}{2}\right)\Big|_0^2 dx\right] = 1$$

$$k\left[\int_0^1 (4x+2)dx\right] = 1$$

$$k\left[2x^2+2x\Big|_0^1\right] = 1$$

$$k(2+2) = 1 \Rightarrow k = \frac{1}{4}$$

Example: Finding Marginal Distributions

Example

Given the joint PDF

$$f(x, y) = 6xy, \quad 0 < x < 1, 0 < y < 1$$

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Solution

$$f_X(x) = \int_0^1 6xy\,dy = 6x\int_0^1 y\,dy = 6x \times \frac{1}{2} = 3x, \quad 0 < x < 1$$

$$f_Y(y) = \int_0^1 6xy\,dx = 6y\int_0^1 x\,dx = 6y \times \frac{1}{2} = 3y, \quad 0 < y < 1$$

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- Defined bivariate continuous random variables.
- Explored joint and marginal distributions.
- Derived conditional distributions and density functions.
- Examined stochastic independence.
- Solved problems on two-dimensional continuous random variables.

Solutions/Answers

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- Problem 1: $k = \frac{1}{4}$
- Problem 2: $f_X(x) = 3x, f_Y(y) = 3y$
- Problem 3: Determine if given $f(x, y)$ satisfies $f(x, y) = f_X(x)f_Y(y)$.

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