Random Variables and Probability Distributions

Data Science and A.I. Lecture Series

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PostNetwork Academy

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- X is a variable quantity behaving randomly, so we call it a random variable.

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  - $F(2) = P(X \leq 2) = P(0) + P(1) + P(2) = 1$

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- The PMF table was shown with overlay effects.
- The CDF gives cumulative probabilities.
- The probabilities sum to 1, ensuring a valid probability distribution.

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