Some Questions of Linear Algebra: Linear Transformation

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 - Additivity: $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ for all $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$.
 - Homogeneity: $T(c\mathbf{v}) = cT(\mathbf{v})$ for all $\mathbf{v} \in \mathbb{R}^n$ and scalars c.

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- Compute T(1,1) = (3,-2) and T(1,-1) = (1,4).
- Express these in terms of the new basis:

$$\begin{bmatrix} 3\\-2 \end{bmatrix} = a \begin{bmatrix} 1\\1 \end{bmatrix} + b \begin{bmatrix} 1\\-1 \end{bmatrix},$$
$$\begin{bmatrix} 1\\4 \end{bmatrix} = c \begin{bmatrix} 1\\1 \end{bmatrix} + d \begin{bmatrix} 1\\-1 \end{bmatrix}.$$

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• Solving, we get $a = \frac{1}{2}, b = \frac{5}{2}, c = \frac{5}{2}, d = -\frac{3}{2}.$

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- Solving, we get $a = \frac{1}{2}, b = \frac{5}{2}, c = \frac{5}{2}, d = -\frac{3}{2}$.
- The transformation matrix in the new basis is:

$$A' = \begin{bmatrix} \frac{1}{2} & \frac{5}{2} \\ \frac{5}{2} & -\frac{3}{2} \end{bmatrix}$$

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Question

Problem Statement:

• Let W be the vector space of all real polynomials of degree at most 3.

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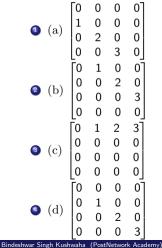
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- Let W be the vector space of all real polynomials of degree at most 3.
- Define $T: W \to W$ by (Tp)(x) = p'(x) where p' is the derivative of p.
- The matrix of T in the basis $\{1,X,X^2,X^3\}$ is given by:



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$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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• The correct option is (b).

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