Some Questions of Linear Algebra: Linear Transformation

Bindeshwar Singh Kushwaha

PostNetwork Academy

Definition: Linear Transformation

- A **linear transformation** $T : \mathbb{R}^n \to \mathbb{R}^m$ is a function that satisfies:

Definition: Linear Transformation

- A **linear transformation** $T : \mathbb{R}^n \to \mathbb{R}^m$ is a function that satisfies:
  - **Additivity:** $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ for all $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$.

Definition: Linear Transformation

- A **linear transformation** $T : \mathbb{R}^n \to \mathbb{R}^m$ is a function that satisfies:
  - **Additivity:** $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ for all $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$.
  - **Homogeneity:** $T(c\mathbf{v}) = cT(\mathbf{v})$ for all $\mathbf{v} \in \mathbb{R}^n$ and scalars c.

Numerical Example: Linear Transformation to Matrix

- Consider $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (2x + y, x - 3y)$.

Numerical Example: Linear Transformation to Matrix

- Consider $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (2x + y, x - 3y)$.
- Consider the basis $\{(1, 1), (1, -1)\}$.

Numerical Example: Linear Transformation to Matrix

- Consider $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (2x + y, x - 3y)$.
- Consider the basis $\{(1, 1), (1, -1)\}$.
- Compute $T(1, 1) = (3, -2)$ and $T(1, -1) = (1, 4)$.

Numerical Example: Linear Transformation to Matrix

- Consider $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (2x + y, x - 3y)$.
- Consider the basis $\{(1, 1), (1, -1)\}$.
- Compute $T(1, 1) = (3, -2)$ and $T(1, -1) = (1, 4)$.
- Express these in terms of the new basis:

$$\begin{bmatrix} 3 \\ -2 \end{bmatrix} = a \begin{bmatrix} 1 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix} = c \begin{bmatrix} 1 \\ 1 \end{bmatrix} + d \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

Numerical Example: Linear Transformation to Matrix

- Consider $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (2x + y, x - 3y)$.
- Consider the basis $\{(1, 1), (1, -1)\}$.
- Compute $T(1, 1) = (3, -2)$ and $T(1, -1) = (1, 4)$.
- Express these in terms of the new basis:

$$\begin{bmatrix} 3 \\ -2 \end{bmatrix} = a \begin{bmatrix} 1 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix} = c \begin{bmatrix} 1 \\ 1 \end{bmatrix} + d \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

- Solving, we get $a = \frac{1}{2}, b = \frac{5}{2}$, $c = \frac{5}{2}, d = -\frac{3}{2}$.

Numerical Example: Linear Transformation to Matrix

- Consider $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (2x + y, x - 3y)$.
- Consider the basis $\{(1, 1), (1, -1)\}$.
- Compute $T(1, 1) = (3, -2)$ and $T(1, -1) = (1, 4)$.
- Express these in terms of the new basis:

$$\begin{bmatrix} 3 \\ -2 \end{bmatrix} = a \begin{bmatrix} 1 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix} = c \begin{bmatrix} 1 \\ 1 \end{bmatrix} + d \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

- Solving, we get $a = \frac{1}{2}, b = \frac{5}{2}, c = \frac{5}{2}, d = -\frac{3}{2}$.
- The transformation matrix in the new basis is:

$$A' = \begin{bmatrix} \frac{1}{2} & \frac{5}{2} \\ \frac{5}{2} & -\frac{3}{2} \end{bmatrix}$$

Question

Problem Statement:

- Let W be the vector space of all real polynomials of degree at most 3.

Question

Problem Statement:

- Let W be the vector space of all real polynomials of degree at most 3.
- Define $T : W \to W$ by $(Tp)(x) = p'(x)$ where p' is the derivative of p.

Question

Problem Statement:

- Let W be the vector space of all real polynomials of degree at most 3.
- Define $T : W \to W$ by $(Tp)(x) = p'(x)$ where p' is the derivative of p.
- The matrix of T in the basis $\{1, X, X^2, X^3\}$ is given by:

1. (a) $$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$
2. (b) $$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
3. (c) $$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
4. (d) $$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

Solution to Question

- The differentiation operator applied to basis polynomials:

Solution to Question

- The differentiation operator applied to basis polynomials:
  - $T(1) = 0, \quad T(X) = 1, \quad T(X^2) = 2X, \quad T(X^3) = 3X^2.$

Solution to Question

- The differentiation operator applied to basis polynomials:
  - $T(1) = 0, \quad T(X) = 1, \quad T(X^2) = 2X, \quad T(X^3) = 3X^2.$
- Expressing these in the basis $\{1, X, X^2, X^3\}$, we derive the transformation matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution to Question

- The differentiation operator applied to basis polynomials:
  - $T(1) = 0, \quad T(X) = 1, \quad T(X^2) = 2X, \quad T(X^3) = 3X^2.$
- Expressing these in the basis $\{1, X, X^2, X^3\}$, we derive the transformation matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- The correct option is (b).

Reach PostNetwork Academy

Website

www.postnetwork.co

Reach PostNetwork Academy

Website

www.postnetwork.co

YouTube Channel

www.youtube.com/@postnetworkacademy

Reach PostNetwork Academy

Website

www.postnetwork.co

YouTube Channel

www.youtube.com/@postnetworkacademy

Facebook Page

www.facebook.com/postnetworkacademy

Reach PostNetwork Academy

Website

www.postnetwork.co

YouTube Channel

www.youtube.com/@postnetworkacademy

Facebook Page

www.facebook.com/postnetworkacademy

LinkedIn Page

www.linkedin.com/company/postnetworkacademy

Thank You!