Law of Total Probability and Examples

Data Science and A.I. Lecture Series

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PostNetwork Academy

Partition of a Sample Space

A set of events E_1, E_2, E_3, E_4 is said to represent a partition of the sample space S if:

- $E_i \cap E_j = \emptyset$ for $i \neq j$, meaning they are pairwise disjoint.

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- $P(E_i) > 0$ for all $i = 1, 2, 3, 4$, meaning each has a nonzero probability.

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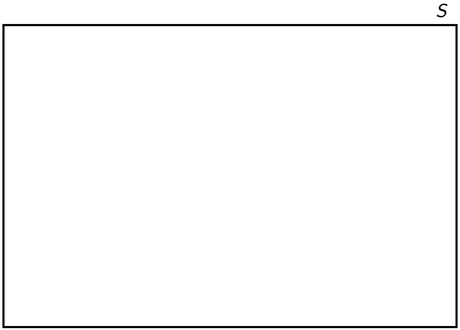
- $P(E_i) > 0$ for all $i = 1, 2, 3, 4$, meaning each has a nonzero probability.
- **Example:**
- Any nonempty event E and its complement E' form a partition since:

$$E \cap E' = \emptyset, \quad E \cup E' = S$$

Diagram: Partition of a Sample Space



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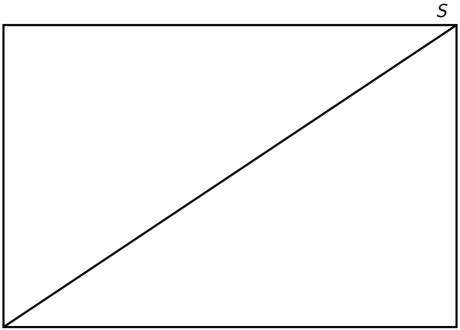

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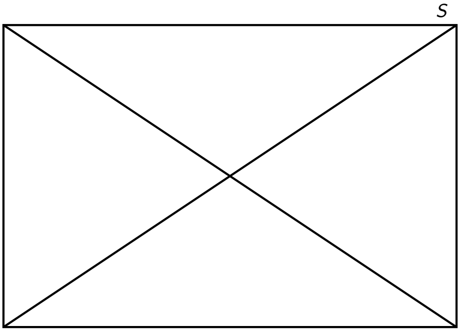

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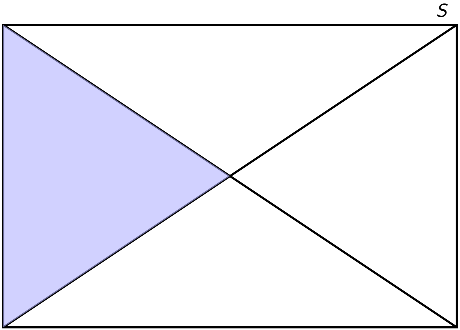

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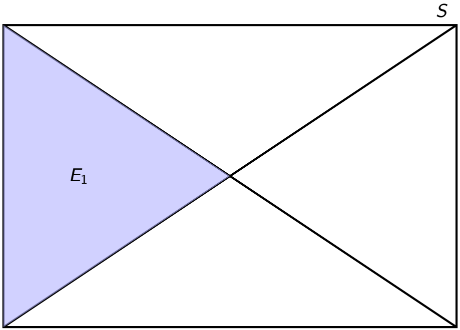

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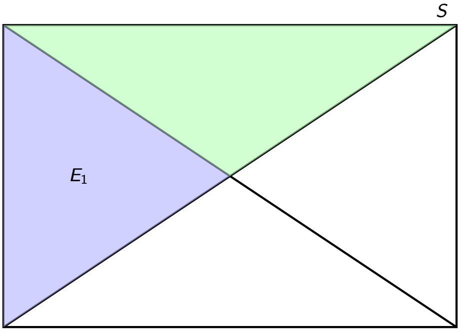

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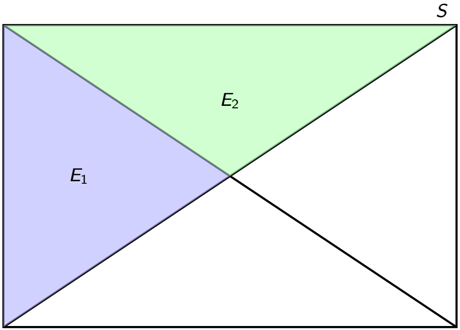

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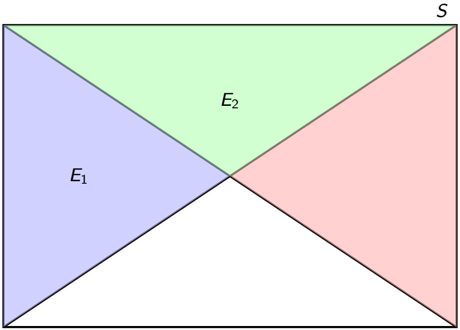

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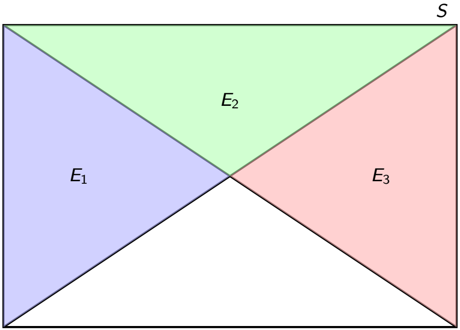

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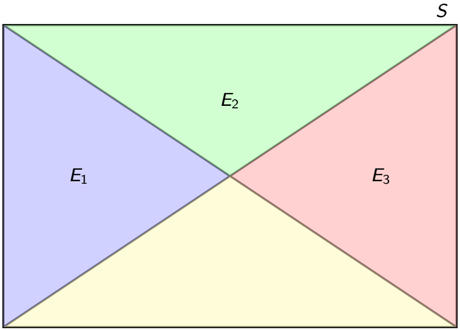

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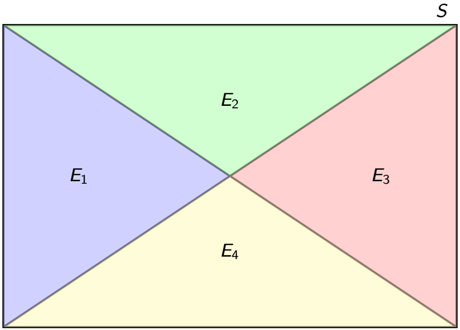

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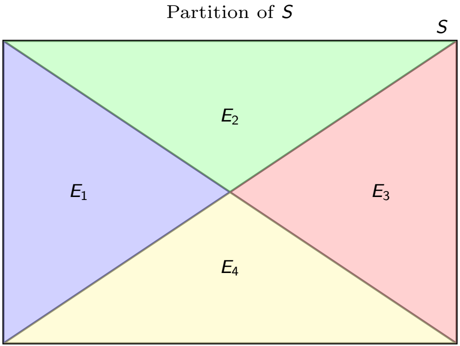

Law of Total Probability

Let E_1, E_2, E_3, E_4 be a partition of the sample space S, where $P(E_i) > 0$ for all i, and let A be any event in S.

Law of Total Probability

Then, the law of total probability states that:

Law of Total Probability

$$P(A) = \sum_{i=1}^{4} P(A|E_i)P(E_i)$$

Law of Total Probability

The steps to derive this result are:

Law of Total Probability

- The events E_1, E_2, E_3, E_4 form a partition of S, meaning they are mutually exclusive and exhaustive.

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- Since one of these events must occur, we express A as:

$$A = (A \cap E_1) \cup (A \cap E_2) \cup (A \cap E_3) \cup (A \cap E_4)$$

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- By the addition rule for probabilities:

$$P(A) = P(A \cap E_1) + P(A \cap E_2) + P(A \cap E_3) + P(A \cap E_4)$$

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- Using the definition of conditional probability, $P(A \cap E_i) = P(A|E_i)P(E_i)$.

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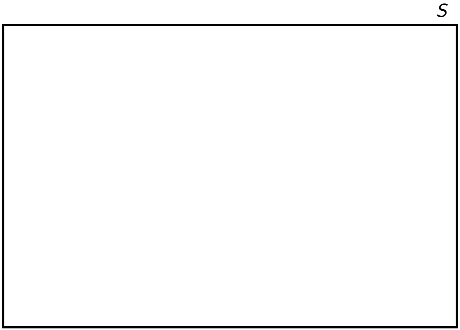
- By the addition rule for probabilities:

$$P(A) = P(A \cap E_1) + P(A \cap E_2) + P(A \cap E_3) + P(A \cap E_4)$$

- Using the definition of conditional probability, $P(A \cap E_i) = P(A|E_i)P(E_i)$.
- Substituting this into the equation gives:

$$P(A) = \sum_{i=1}^{4} P(A|E_i)P(E_i)$$

Diagram: Law of Total Probability



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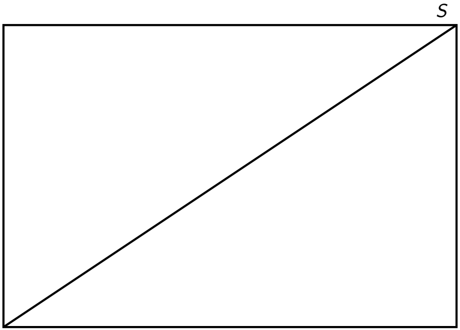

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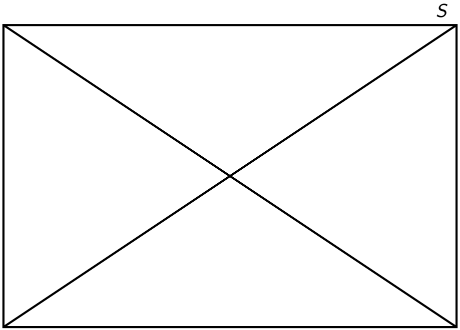

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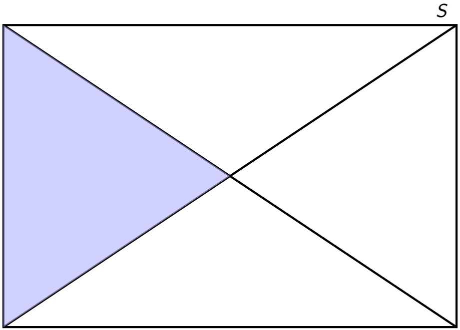

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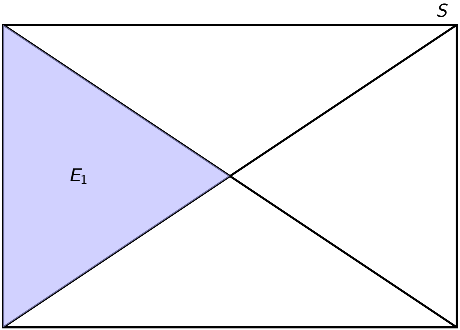

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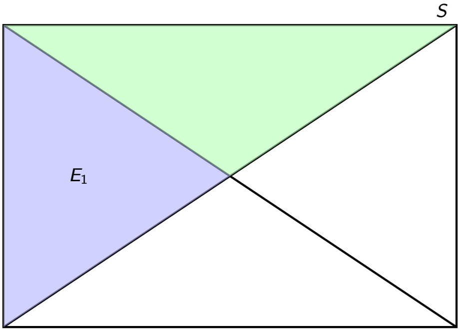

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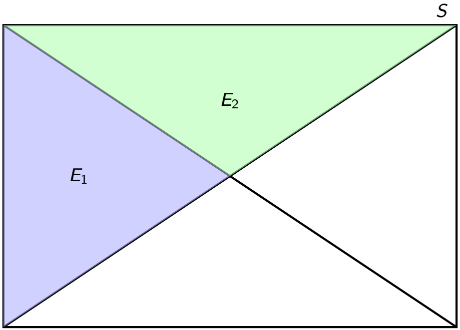

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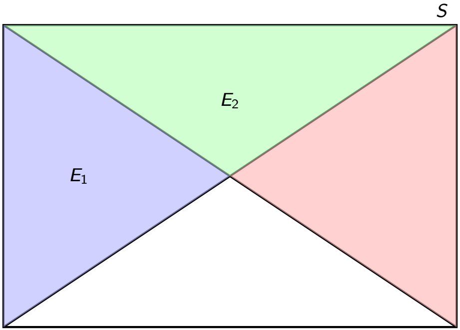

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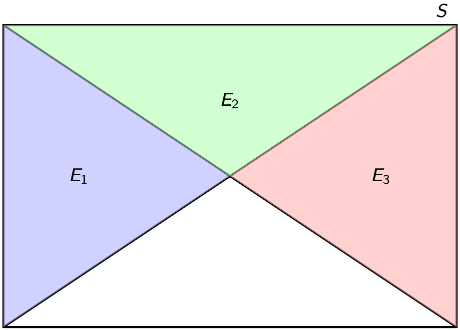

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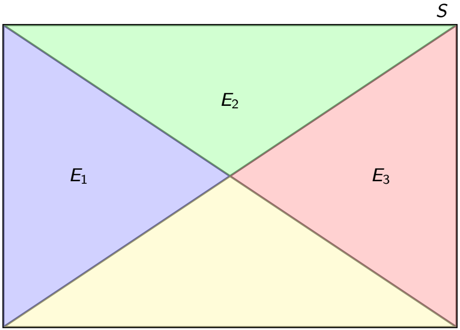

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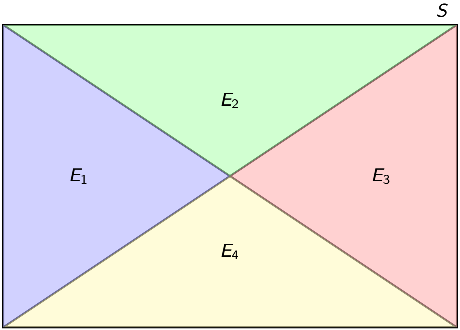

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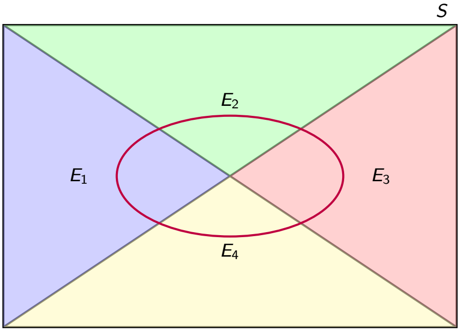

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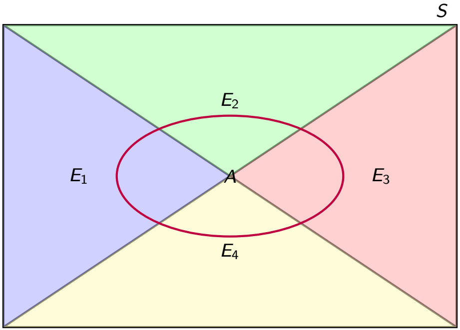
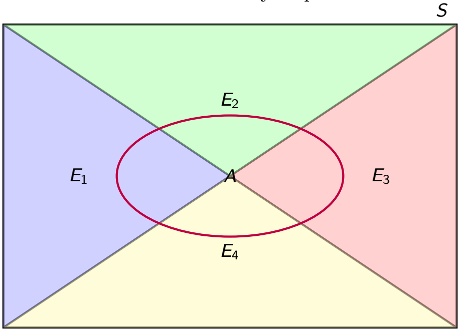


Diagram: Law of Total Probability

Law of Total Probability Representation



Example: Probability of Drawing a Red or White Ball

Question:

- There are two bags:
  - First bag: 5 red, 6 white balls
  - Second bag: 3 red, 4 white balls
- One bag is selected at random, and a ball is drawn.
- Find the probability that the ball is:
  - (i) Red
  - (ii) White

Solution to Example 1

Question (Reiterated):

- There are two bags:
  - First bag: 5 red, 6 white balls
  - Second bag: 3 red, 4 white balls
- One bag is selected at random, and a ball is drawn.
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Solution:

- Define events:

Solution to Example 1

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Solution:

- Define events:
  - E_1: Selecting the first bag, $P(E_1) = 1/2$

Solution to Example 1

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Solution:

- Define events:
  - E_1: Selecting the first bag, $P(E_1) = 1/2$
  - E_2: Selecting the second bag, $P(E_2) = 1/2$

Solution to Example 1

Question (Reiterated):

- There are two bags:
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  - Second bag: 3 red, 4 white balls
- One bag is selected at random, and a ball is drawn.
- Find the probability that the ball is:
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  - (ii) White

Solution:

- Define events:
  - E_1: Selecting the first bag, $P(E_1) = 1/2$
  - E_2: Selecting the second bag, $P(E_2) = 1/2$
- Conditional probabilities:

Solution to Example 1

Question (Reiterated):

- There are two bags:
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- One bag is selected at random, and a ball is drawn.
- Find the probability that the ball is:
  - (i) Red
  - (ii) White

Solution:

- Define events:
  - E_1: Selecting the first bag, $P(E_1) = 1/2$
  - E_2: Selecting the second bag, $P(E_2) = 1/2$
- Conditional probabilities:
  - $P(R|E_1) = 5/11$, $P(W|E_1) = 6/11$

Solution to Example 1

Question (Reiterated):

- There are two bags:
  - First bag: 5 red, 6 white balls
  - Second bag: 3 red, 4 white balls
- One bag is selected at random, and a ball is drawn.
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Solution:

- Define events:
  - E_1: Selecting the first bag, $P(E_1) = 1/2$
  - E_2: Selecting the second bag, $P(E_2) = 1/2$
- Conditional probabilities:
  - $P(R|E_1) = 5/11$, $P(W|E_1) = 6/11$
  - $P(R|E_2) = 3/7$, $P(W|E_2) = 4/7$

Solution to Example 1

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- There are two bags:
  - First bag: 5 red, 6 white balls
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Solution:

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  - E_1: Selecting the first bag, $P(E_1) = 1/2$
  - E_2: Selecting the second bag, $P(E_2) = 1/2$
- Conditional probabilities:
  - $P(R|E_1) = 5/11$, $P(W|E_1) = 6/11$
  - $P(R|E_2) = 3/7$, $P(W|E_2) = 4/7$
- Using the law of total probability:

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Solution:

- Define events:
  - E_1: Selecting the first bag, $P(E_1) = 1/2$
  - E_2: Selecting the second bag, $P(E_2) = 1/2$
- Conditional probabilities:
  - $P(R|E_1) = 5/11$, $P(W|E_1) = 6/11$
  - $P(R|E_2) = 3/7$, $P(W|E_2) = 4/7$
- Using the law of total probability:
  - $P(R) = (1/2 \times 5/11) + (1/2 \times 3/7) = 34/77$

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- One bag is selected at random, and a ball is drawn.
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Solution:

- Define events:
  - E_1: Selecting the first bag, $P(E_1) = 1/2$
  - E_2: Selecting the second bag, $P(E_2) = 1/2$
- Conditional probabilities:
  - $P(R|E_1) = 5/11$, $P(W|E_1) = 6/11$
  - $P(R|E_2) = 3/7$, $P(W|E_2) = 4/7$
- Using the law of total probability:
  - $P(R) = (1/2 \times 5/11) + (1/2 \times 3/7) = 34/77$
  - $P(W) = (1/2 \times 6/11) + (1/2 \times 4/7) = 43/77$

Example 2: Probability of Selecting a Defective Item

Question:

- A factory has three machines producing outputs:
  - Machine X: 3000 units (1% defective)
  - Machine Y: 2500 units (1.2% defective)
  - Machine Z: 4500 units (2% defective)
- An item is drawn at random.
- Find the probability that it is defective.

Solution to Example 2

Question (Reiterated):

- A factory has three machines producing outputs:
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- An item is drawn at random.
- Find the probability that it is defective.

Solution:

- Define events:
  - $P(E_X) = 3000/10000$, $P(E_Y) = 2500/10000$, $P(E_Z) = 4500/10000$

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Question (Reiterated):

- A factory has three machines producing outputs:
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Solution:

- Define events:
  - $P(E_X) = 3000/10000,\ P(E_Y) = 2500/10000,\ P(E_Z) = 4500/10000$
- Conditional probabilities:

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Question (Reiterated):

- A factory has three machines producing outputs:
  - Machine X: 3000 units (1% defective)
  - Machine Y: 2500 units (1.2% defective)
  - Machine Z: 4500 units (2% defective)
- An item is drawn at random.
- Find the probability that it is defective.

Solution:

- Define events:
  - $P(E_X) = 3000/10000$, $P(E_Y) = 2500/10000$, $P(E_Z) = 4500/10000$
- Conditional probabilities:
  - $P(D|E_X) = 0.01$, $P(D|E_Y) = 0.012$, $P(D|E_Z) = 0.02$

Solution to Example 2

Question (Reiterated):

- A factory has three machines producing outputs:
  - Machine X: 3000 units (1% defective)
  - Machine Y: 2500 units (1.2% defective)
  - Machine Z: 4500 units (2% defective)
- An item is drawn at random.
- Find the probability that it is defective.

Solution:

- Define events:
  - $P(E_X) = 3000/10000,\ P(E_Y) = 2500/10000,\ P(E_Z) = 4500/10000$
- Conditional probabilities:
  - $P(D|E_X) = 0.01,\ P(D|E_Y) = 0.012,\ P(D|E_Z) = 0.02$
- Using the law of total probability:

Solution to Example 2

Question (Reiterated):

- A factory has three machines producing outputs:
  - Machine X: 3000 units (1% defective)
  - Machine Y: 2500 units (1.2% defective)
  - Machine Z: 4500 units (2% defective)
- An item is drawn at random.
- Find the probability that it is defective.

Solution:

- Define events:
  - $P(E_X) = 3000/10000,\ P(E_Y) = 2500/10000,\ P(E_Z) = 4500/10000$
- Conditional probabilities:
  - $P(D|E_X) = 0.01,\ P(D|E_Y) = 0.012,\ P(D|E_Z) = 0.02$
- Using the law of total probability:
  - $P(D) = (0.3 \times 0.01) + (0.25 \times 0.012) + (0.45 \times 0.02) = 0.015$

Example 3: Probability of Getting a Head

Question:

- There are two coins:
  - One unbiased coin
  - One two-headed coin
- A coin is selected at random and tossed.
- Find the probability of getting a head.

Solution to Example 3

Question (Reiterated):

- There are two coins:
  - One unbiased coin
  - One two-headed coin
- A coin is selected at random and tossed.
- Find the probability of getting a head.

Solution:

- Define events:

Solution to Example 3

Question (Reiterated):

- There are two coins:
  - One unbiased coin
  - One two-headed coin
- A coin is selected at random and tossed.
- Find the probability of getting a head.

Solution:

- Define events:
  - E_1: Selecting unbiased coin, $P(E_1) = 1/2$

Solution to Example 3

Question (Reiterated):

- There are two coins:
  - One unbiased coin
  - One two-headed coin
- A coin is selected at random and tossed.
- Find the probability of getting a head.

Solution:

- Define events:
  - E_1: Selecting unbiased coin, $P(E_1) = 1/2$
  - E_2: Selecting two-headed coin, $P(E_2) = 1/2$

Solution to Example 3

Question (Reiterated):

- There are two coins:
  - One unbiased coin
  - One two-headed coin
- A coin is selected at random and tossed.
- Find the probability of getting a head.

Solution:

- Define events:
  - E_1: Selecting unbiased coin, $P(E_1) = 1/2$
  - E_2: Selecting two-headed coin, $P(E_2) = 1/2$
- Conditional probabilities:

Solution to Example 3

Question (Reiterated):

- There are two coins:
  - One unbiased coin
  - One two-headed coin
- A coin is selected at random and tossed.
- Find the probability of getting a head.

Solution:

- Define events:
  - E_1: Selecting unbiased coin, $P(E_1) = 1/2$
  - E_2: Selecting two-headed coin, $P(E_2) = 1/2$
- Conditional probabilities:
  - $P(H|E_1) = 1/2$, $P(H|E_2) = 1$

Solution to Example 3

Question (Reiterated):

- There are two coins:
  - One unbiased coin
  - One two-headed coin
- A coin is selected at random and tossed.
- Find the probability of getting a head.

Solution:

- Define events:
  - E_1: Selecting unbiased coin, $P(E_1) = 1/2$
  - E_2: Selecting two-headed coin, $P(E_2) = 1/2$
- Conditional probabilities:
  - $P(H|E_1) = 1/2$, $P(H|E_2) = 1$
- Using law of total probability:

Solution to Example 3

Question (Reiterated):

- There are two coins:
  - One unbiased coin
  - One two-headed coin
- A coin is selected at random and tossed.
- Find the probability of getting a head.

Solution:

- Define events:
  - E_1: Selecting unbiased coin, $P(E_1) = 1/2$
  - E_2: Selecting two-headed coin, $P(E_2) = 1/2$
- Conditional probabilities:
  - $P(H|E_1) = 1/2$, $P(H|E_2) = 1$
- Using law of total probability:
  - $P(H) = (1/2 \times 1/2) + (1/2 \times 1) = 3/4$

Example 4: Probability of Introducing Co-Education

Question:

- Three persons are considered for a principal position, with selection probabilities:
  - Person 1: $4/9$
  - Person 2: $3/9$
  - Person 3: $2/9$
- Probability of introducing co-education:
  - Person 1: 0.2
  - Person 2: 0.3
  - Person 3: 0.5
- Find the probability that co-education is introduced in the college.

Solution to Example 4

Question (Reiterated):

- Three persons are considered for a principal position, with selection probabilities:
  - Person 1: 4/9
  - Person 2: 3/9
  - Person 3: 2/9
- Probability of introducing co-education:
  - Person 1: 0.2
  - Person 2: 0.3
  - Person 3: 0.5
- Find the probability that co-education is introduced in the college.

Solution:

- Using law of total probability:

Solution to Example 4

Question (Reiterated):

- Three persons are considered for a principal position, with selection probabilities:
  - Person 1: 4/9
  - Person 2: 3/9
  - Person 3: 2/9
- Probability of introducing co-education:
  - Person 1: 0.2
  - Person 2: 0.3
  - Person 3: 0.5
- Find the probability that co-education is introduced in the college.

Solution:

- Using law of total probability:
  - $P(A) = (4/9 \times 0.2) + (3/9 \times 0.3) + (2/9 \times 0.5) = 0.3$

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