Discrete Random Variable and Probability Mass Function

Data Science and A.I. Lecture Series

Bindeshwar Singh Kushwaha

PostNetwork Academy

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- Example: Number of students present each day in a class.

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  - $\sum_i p(x_i) = 1$.

Example 1: Valid Probability Distribution?

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X	$P(X)$
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- Sum: $\frac{1}{2} + \frac{3}{4} = \frac{5}{4} > 1$.
- **Not** a probability distribution since the sum exceeds 1.

Example 2: Probability Distribution

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- Solving: $8c = 1 \Rightarrow c = \frac{1}{8}$.

Example 3: Number of Heads in 3 Coin Tosses

Question: Find the probability distribution of the number of heads when tossing 3 fair coins.

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- Examples illustrated the application of PMFs.

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