Bayes' Theorem and Examples

Data Science and A.I. Lecture Series

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Bayes' Theorem

Formula:

$$P(E_i|A) = \frac{P(E_i)P(A|E_i)}{\sum_{j=1}^{n} P(E_j)P(A|E_j)}$$

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- Helps compute reverse probabilities.
- Useful when we have prior probabilities and conditional probabilities.
- Often used in medical testing, reliability analysis, and decision-making.

Bayes' Theorem: Key Terminology

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- The conditional probability $P(E_i|A)$ is called the **posterior probability** of hypothesis E_i.

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- Bayes' theorem is also known as the **formula for the probability of "causes"**.
- The events E_i form a **partition** of the sample space S, meaning one and only one of them must occur.
- Hence, the theorem gives us the probability of a particular cause E_i given that event A has occurred.

Example: Probability from Bag II

Problem:

- Bag I: 3 red, 4 black balls.

Example: Probability from Bag II

Problem:

- Bag I: 3 red, 4 black balls.
- Bag II: 5 red, 6 black balls.

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- Bag I: 3 red, 4 black balls.
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- A bag is chosen randomly, and a red ball is drawn.
- Find the probability that it was drawn from Bag II.

Example: Step 1 - Given Data

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Step 1: Define Probabilities

- Probability of choosing Bag I: $P(B_1) = \frac{1}{2}$

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- Probability of choosing Bag I: $P(B_1) = \frac{1}{2}$
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- Probability of choosing Bag I: $P(B_1) = \frac{1}{2}$
- Probability of choosing Bag II: $P(B_2) = \frac{1}{2}$
- Probability of drawing red from Bag I: $P(R|B_1) = \frac{3}{7}$

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- Probability of drawing red from Bag II: $P(R|B_2) = \frac{5}{11}$

Example: Step 2 - Apply Bayes' Theorem

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Step 2: Apply Bayes' Formula

$$P(B_2|R) = \frac{P(B_2)P(R|B_2)}{P(B_1)P(R|B_1) + P(B_2)P(R|B_2)}$$

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$$= \frac{5}{8}$$

Example: Probability of Another Gold Coin

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- Box I: 2 gold coins.

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Problem:

- Box I: 2 gold coins.
- Box II: 2 silver coins.

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- Box I: 2 gold coins.
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- A box is chosen randomly, and a gold coin is drawn.
- Find the probability that the other coin in the box is also gold.

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Step 1: Assign Probabilities

- $P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$
- $P(G|B_1) = 1, P(G|B_2) = 0, P(G|B_3) = \frac{1}{2}$

Example: Step 2 - Apply Bayes' Theorem

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$$= \frac{2}{3}$$

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