Set Operations and Important Laws

Data Science and A.I. Lecture Series

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Set Operations: Union

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$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

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$$A = \{1, 2, 3\}, B = \{3, 4, 5\} \implies A \cup B = \{1, 2, 3, 4, 5\}$$

- Example 2:

$$A = \{a, b\}, B = \{b, c, d\} \implies A \cup B = \{a, b, c, d\}$$

Set Operations: Intersection

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$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

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- Example 2:

$$A = \{a, b, c\}, B = \{b, c, d\} \implies A \cap B = \{b, c\}$$

Set Operations: Complement

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$$U = \{1, 2, 3, 4, 5\}, A = \{1, 2\} \implies A^c = \{3, 4, 5\}$$

Set Operations: Complement

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- Example 2:

$$U = \{a, b, c, d\}, A = \{a, b\} \implies A^c = \{c, d\}$$

Set Operations: Difference

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$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

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$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

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$$A = \{1, 2, 3\}, B = \{3, 4, 5\} \implies A - B = \{1, 2\}$$

- Example 2:

$$A = \{a, b, c\}, B = \{b, c, d\} \implies A - B = \{a\}$$

Set Operations: Symmetric Difference

- Definition:

$$A \Delta B = (A - B) \cup (B - A)$$

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- Example:

$$A = \{1, 2, 3\}, B = \{3, 4, 5\} \implies A \Delta B = \{1, 2, 4, 5\}$$

Idempotent and Identity Laws

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- Example:

$$A = \{1, 2\}, U = \{1, 2, 3\}, \emptyset = \{\}$$

$$A \cup \emptyset = \{1, 2\}, \quad A \cap U = \{1, 2\}$$

Commutative and Associative Laws

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Commutative and Associative Laws

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$$A \cup B = B \cup A, \quad A \cap B = B \cap A$$

- Associative Laws:

$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$

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$$(A \cap B) \cap C = A \cap (B \cap C)$$

- Example:

$$A = \{1, 2\}, B = \{2, 3\}, C = \{3, 4\}$$

$$(A \cup B) \cup C = \{1, 2, 3, 4\}, \quad A \cup (B \cup C) = \{1, 2, 3, 4\}$$

Distributive and De Morgan's Laws

- Distributive Laws:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Distributive and De Morgan's Laws

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- De Morgan's Laws:

$$(A \cup B)^c = A^c \cap B^c$$
$$(A \cap B)^c = A^c \cup B^c$$

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$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
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- De Morgan's Laws:
$$(A \cup B)^c = A^c \cap B^c$$
$$(A \cap B)^c = A^c \cup B^c$$
- Example: Verify $(A \cap B)^c = A^c \cup B^c$ for:
$$A = \{1, 2\}, B = \{2, 3\}, U = \{1, 2, 3, 4\}$$
$$A^c = \{3, 4\}, B^c = \{1, 4\}, A^c \cup B^c = \{1, 3, 4\}$$

Applications of Sets

Venn diagrams are helpful in establishing many important relations between different sets. These relations are useful in solving practical problems. Some key formulas include:

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

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- $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$

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- $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$
- $n(A - B) = n(A) - n(A \cap B)$
- $n(B - A) = n(B) - n(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

Example: Language Proficiency

In a group of 500 persons, 400 can speak Hindi and 150 can speak English. Find:

- (i) the number of persons who can speak both Hindi and English

Example: Language Proficiency

In a group of 500 persons, 400 can speak Hindi and 150 can speak English. Find:

- (i) the number of persons who can speak both Hindi and English
- (ii) the number of persons who can speak only Hindi

Example: Language Proficiency

In a group of 500 persons, 400 can speak Hindi and 150 can speak English. Find:

- (i) the number of persons who can speak both Hindi and English
- (ii) the number of persons who can speak only Hindi
- (iii) the number of persons who can speak only English

Solution

Let A and B denote the sets of persons who can speak Hindi and English, respectively. The given information is:

$$n(A \cup B) = 500, \quad n(A) = 400, \quad n(B) = 150$$

- (i) Using $n(A \cup B) = n(A) + n(B) - n(A \cap B)$,

$$500 = 400 + 150 - n(A \cap B)$$

$$n(A \cap B) = 50$$

The number of persons who can speak both Hindi and English is 50.

Solution

Let A and B denote the sets of persons who can speak Hindi and English, respectively. The given information is:

$$n(A \cup B) = 500, \quad n(A) = 400, \quad n(B) = 150$$

- (i) Using $n(A \cup B) = n(A) + n(B) - n(A \cap B)$,

$$500 = 400 + 150 - n(A \cap B)$$

$$n(A \cap B) = 50$$

  The number of persons who can speak both Hindi and English is 50.
- (ii) Using $n(A - B) = n(A) - n(A \cap B)$,

$$n(A - B) = 400 - 50 = 350$$

  The number of persons who can speak only Hindi is 350.

Solution

Let A and B denote the sets of persons who can speak Hindi and English, respectively. The given information is:

$$n(A \cup B) = 500, \quad n(A) = 400, \quad n(B) = 150$$

- (i) Using $n(A \cup B) = n(A) + n(B) - n(A \cap B)$,

$$500 = 400 + 150 - n(A \cap B)$$

$$n(A \cap B) = 50$$

  The number of persons who can speak both Hindi and English is 50.
- (ii) Using $n(A - B) = n(A) - n(A \cap B)$,

$$n(A - B) = 400 - 50 = 350$$

  The number of persons who can speak only Hindi is 350.
- (iii) Using $n(B - A) = n(B) - n(A \cap B)$,

$$n(B - A) = 150 - 50 = 100$$

  The number of persons who can speak only English is 100.

Summary of Results

- Number of persons who can speak both Hindi and English: 50

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