Probability Problems Based on Classical Definition of Probability

Data Science and A.I. Lecture Series

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Questions

- What is the total number of outcomes (sample space)?

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- How do we determine favorable cases?

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- How do we determine favorable cases?
- How do probability rules apply to the problem?

Example: Throwing Two Dice

Find the probability of:

1. A doublet (same number on both dice)

Example: Throwing Two Dice

Find the probability of:

1. A doublet (same number on both dice)
2. Sum equal to 7

Example: Throwing Two Dice

Find the probability of:

1. A doublet (same number on both dice)
2. Sum equal to 7
3. Sum greater than 8

Example: Throwing Two Dice

Find the probability of:

1. A doublet (same number on both dice)
2. Sum equal to 7
3. Sum greater than 8
4. 3 on the first die and a multiple of 2 on the second die

Example: Throwing Two Dice

Find the probability of:

1. A doublet (same number on both dice)
2. Sum equal to 7
3. Sum greater than 8
4. 3 on the first die and a multiple of 2 on the second die
5. Prime number on the first die and odd prime on the second die

Example: Throwing Two Dice

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1. A doublet (same number on both dice)
2. Sum equal to 7
3. Sum greater than 8
4. 3 on the first die and a multiple of 2 on the second die
5. Prime number on the first die and odd prime on the second die

Solution:

- *(i)* Favorable outcomes: {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

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- *(iii)* Favorable outcomes: {(3,6), (4,5), (5,4), (6,3), (4,6), (5,5), (5,6), (6,5), (6,6)}

$$P(C) = \frac{10}{36} = \frac{5}{18}$$

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- *(iv)* Favorable outcomes: {(3,2), (3,4), (3,6)}

$$P(D) = \frac{3}{36} = \frac{1}{12}$$

Example: Throwing Two Dice

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1. A doublet (same number on both dice)
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Solution:

- *(i)* Favorable outcomes: {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}

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- *(ii)* Favorable outcomes: {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}

$$P(B) = \frac{6}{36} = \frac{1}{6}$$

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- *(iv)* Favorable outcomes: {(3,2), (3,4), (3,6)}

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- *(v)* Favorable outcomes: {(2,3), (2,5), (3,3), (3,5), (5,3), (5,5)}

$$P(E) = \frac{6}{36} = \frac{1}{6}$$

Example: Drawing a Card and find the probability of

1. Drawing a red card

Example: Drawing a Card and find the probability of

1. Drawing a red card
2. Drawing a face card

Example: Drawing a Card and find the probability of

1. Drawing a red card
2. Drawing a face card
3. Drawing a spade card

Example: Drawing a Card and find the probability of

1. Drawing a red card
2. Drawing a face card
3. Drawing a spade card
4. Drawing a card other than clubs

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1. Drawing a red card
2. Drawing a face card
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5. Drawing a king

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1. Drawing a red card
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Solution:

- *(i)* Red cards: 26

$$P(A) = \frac{26}{52} = \frac{1}{2}$$

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1. Drawing a red card
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Solution:

- *(i)* Red cards: 26

$$P(A) = \frac{26}{52} = \frac{1}{2}$$

- *(ii)* Face cards: 12

$$P(B) = \frac{12}{52} = \frac{3}{13}$$

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$$P(C) = \frac{13}{52} = \frac{1}{4}$$

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$$P(C) = \frac{13}{52} = \frac{1}{4}$$

- *(iv)* Non-club cards: 39

$$P(D) = \frac{39}{52} = \frac{3}{4}$$

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2. Drawing a face card
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$$P(C) = \frac{13}{52} = \frac{1}{4}$$

- *(iv)* Non-club cards: 39

$$P(D) = \frac{39}{52} = \frac{3}{4}$$

- *(v)* Kings: 4

$$P(E) = \frac{4}{52} = \frac{1}{13}$$

Example: Two Children

Find the probability of:

1. Elder child is a girl

Example: Two Children

Find the probability of:

1. Elder child is a girl
2. Younger child is a girl

Example: Two Children

Find the probability of:

1. Elder child is a girl
2. Younger child is a girl
3. Both are girls

Example: Two Children

Find the probability of:

1. Elder child is a girl
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4. Both are of opposite sexes

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Solution:

- *(i)* Favorable outcomes: GG, GB

$$P(A) = \frac{2}{4} = \frac{1}{2}$$

Example: Two Children

Find the probability of:

1. Elder child is a girl
2. Younger child is a girl
3. Both are girls
4. Both are of opposite sexes

Solution:

- *(i)* Favorable outcomes: GG, GB

$$P(A) = \frac{2}{4} = \frac{1}{2}$$

- *(ii)* Favorable outcomes: GG, BG

$$P(B) = \frac{2}{4} = \frac{1}{2}$$

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$$P(C) = \frac{1}{4}$$

Example: Two Children

Find the probability of:

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Solution:

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$$P(D) = \frac{2}{4} = \frac{1}{2}$$

Example: 53 Sundays in a Non-Leap Year

Find the probability of getting 53 Sundays.

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Find the probability of getting 53 Sundays. Solution:

- Number of days in a non-leap year: 365

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Example: 53 Sundays in a Non-Leap Year

Find the probability of getting 53 Sundays. Solution:

- Number of days in a non-leap year: 365
- Excess day options: {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}
- Favorable outcome: Sunday

$$P(A) = \frac{1}{7}$$

Example: Letters in "STATISTICS"

What is the probability of choosing a vowel?

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What is the probability of choosing a vowel? Solution:

- Total letters: 10

Example: Letters in "STATISTICS"

What is the probability of choosing a vowel? Solution:

- Total letters: 10
- Vowels: A, I, I (3 vowels)

$$P(A) = \frac{3}{10}$$

Example: Horse Racing

Three horses A, B, and C are in a race. A is twice as likely to win as B, and B is twice as likely to win as C. Find their respective probabilities.

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- $P(B) = 2p$, $P(A) = 4p$

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- Let $P(C) = p$
- $P(B) = 2p,\ P(A) = 4p$
- Using $P(A) + P(B) + P(C) = 1$:

$$4p + 2p + p = 1$$

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- Solve for p:

$$7p = 1 \Rightarrow p = \frac{1}{7}$$

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- Solve for p:

$$7p = 1 \Rightarrow p = \frac{1}{7}$$

- Final probabilities:

$$P(A) = \frac{4}{7},\ P(B) = \frac{2}{7},\ P(C) = \frac{1}{7}$$

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