Examples and Theorem Related to Combinations

Data Science and A.I. Lecture Series

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Theorem: Relationship Between Permutations and Combinations

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The number of permutations of n different objects taken r at a time is related to the number of combinations by:

$$P_r^n = C_r^n \cdot r!$$

where $0 < r \leq n$.

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- A combination C_r^n counts the number of ways to select r objects from n, without considering order.
- A permutation P_r^n includes all possible arrangements of the selected r objects.
- Therefore, multiplying C_r^n by $r!$, the number of ways to arrange r objects, gives P_r^n.

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- The right-hand side C_r^{n+1} represents choosing r items from $n+1$, accounting for all cases.

Example: Solving a Combination Equation

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If $C_9^n = C_8^n$, find C_n^{17}.

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- Therefore:

$$C_n^{17} = C_{17}^{17} = 1$$

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  - The total combinations consider all possible selections.
  - The second calculation focuses on specific groups (1 man and 2 women).

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- Explanation:
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  - Using the combination formula ensures all cases are covered systematically.

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