Conditional Probability and Multiplicative Law, Independent Events

Data Science and A.I. Lecture Series

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Conditional Probability

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- Example from a deck of cards:
  - Let A be the event "drawing a black face card".
  - Let B be the event "drawing a spade".
  - If B occurs, then $P(A|B) = \frac{3}{13}$, since there are 3 black face cards in 13 spades.

Multiplicative Law of Probability

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- If A and B are independent:
$$P(A \cap B) = P(A) \cdot P(B).$$
- For three events A, B, and C:
$$P(A \cap B \cap C) = P(A) \cdot P(B|A) \cdot P(C|A \cap B).$$

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- $P(A) = \frac{3}{6}, \quad P(B) = \frac{3}{6}, \quad P(A \cap B) = \frac{1}{6}$.
- Required probability:

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}.$$

Example 2: Probability of Boys in a Family

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$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}.$$

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Solution:

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- Case (iii): D: Event "at least one child is a boy" $D = \{BB, BG, GB\}$.

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Solution:

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- Let A: Event "both children are boys" $A = \{BB\}$.
- Case (i): B: Event "younger child is a boy" $B = \{BB, BG\}$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}.$$

- Case (ii): C: Event "older child is a boy" $C = \{BB, GB\}$.

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}.$$

- Case (iii): D: Event "at least one child is a boy" $D = \{BB, BG, GB\}$.

$$P(A|D) = \frac{P(A \cap D)}{P(D)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}.$$

Example 3: Drawing Balls Without Replacement

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Solution:

- Let A: Event "first ball is red", $P(A) = \frac{4}{11}$.
- Let $B|A$: Event "second ball is red given the first ball is red", $P(B|A) = \frac{3}{10}$.

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- Let $B|A$: Event "second ball is red given the first ball is red", $P(B|A) = \frac{3}{10}$.
- Required probability:

$$P(A \cap B) = P(A) \cdot P(B|A) = \frac{4}{11} \cdot \frac{3}{10} = \frac{12}{110} = \frac{6}{55}.$$

Example 4: Drawing Cards

Problem: Three cards are drawn one by one without replacement from a well-shuffled deck of 52 cards. Find the probability that:

- The first card is a Jack.

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Problem: Three cards are drawn one by one without replacement from a well-shuffled deck of 52 cards. Find the probability that:

- The first card is a Jack.
- The second card is a Queen.

Example 4: Drawing Cards

Problem: Three cards are drawn one by one without replacement from a well-shuffled deck of 52 cards. Find the probability that:

- The first card is a Jack.
- The second card is a Queen.
- The third card is again a Jack.

Example 4: Drawing Cards

Problem: Three cards are drawn one by one without replacement from a well-shuffled deck of 52 cards. Find the probability that:

- The first card is a Jack.
- The second card is a Queen.
- The third card is again a Jack.

Solution:

- Let E_1: Event "first card is a Jack" $P(E_1) = \frac{4}{52}$.

Example 4: Drawing Cards

Problem: Three cards are drawn one by one without replacement from a well-shuffled deck of 52 cards. Find the probability that:

- The first card is a Jack.
- The second card is a Queen.
- The third card is again a Jack.

Solution:

- Let E_1: Event "first card is a Jack" $P(E_1) = \frac{4}{52}$.
- Let $E_2|E_1$: Event "second card is a Queen, given first card is a Jack" $P(E_2|E_1) = \frac{4}{51}$.

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Problem: Three cards are drawn one by one without replacement from a well-shuffled deck of 52 cards. Find the probability that:

- The first card is a Jack.
- The second card is a Queen.
- The third card is again a Jack.

Solution:

- Let E_1: Event "first card is a Jack" $P(E_1) = \frac{4}{52}$.
- Let $E_2|E_1$: Event "second card is a Queen, given first card is a Jack" $P(E_2|E_1) = \frac{4}{51}$.
- Let $E_3|E_1 \cap E_2$: Event "third card is a Jack, given the first two events occurred" $P(E_3|E_1 \cap E_2) = \frac{3}{50}$.

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Solution:

- Let E_1: Event "first card is a Jack" $P(E_1) = \frac{4}{52}$.
- Let $E_2|E_1$: Event "second card is a Queen, given first card is a Jack" $P(E_2|E_1) = \frac{4}{51}$.
- Let $E_3|E_1 \cap E_2$: Event "third card is a Jack, given the first two events occurred" $P(E_3|E_1 \cap E_2) = \frac{3}{50}$.
- Required probability:

$$P(E_1 \cap E_2 \cap E_3) = P(E_1) \cdot P(E_2|E_1) \cdot P(E_3|E_1 \cap E_2)$$

$$= \frac{4}{52} \cdot \frac{4}{51} \cdot \frac{3}{50} = \frac{48}{132600} = \frac{2}{5525}.$$

Example 5: Selecting a Rich Brilliant Boy

Problem: A class has 50 students: 10 boys and 40 girls. Five students are rich, and 15 students are brilliant. What is the probability of selecting a brilliant rich boy?

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- Let B: Event "student is rich" $P(B) = \frac{5}{50}$.

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Problem: A class has 50 students: 10 boys and 40 girls. Five students are rich, and 15 students are brilliant. What is the probability of selecting a brilliant rich boy?
Solution:

- Let A: Event "student is brilliant" $P(A) = \frac{15}{50}$.
- Let B: Event "student is rich" $P(B) = \frac{5}{50}$.
- Let C: Event "student is a boy" $P(C) = \frac{10}{50}$.

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Solution:

- Let A: Event "student is brilliant" $P(A) = \frac{15}{50}$.
- Let B: Event "student is rich" $P(B) = \frac{5}{50}$.
- Let C: Event "student is a boy" $P(C) = \frac{10}{50}$.
- Assuming independence, the required probability:

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C) = \frac{15}{50} \cdot \frac{5}{50} \cdot \frac{10}{50} = \frac{3}{500}.$$

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Thank You!