Axiomatic Approach to Probability

Data Science and A.I. Lecture Series

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- Let A be an event that is a subset of S.
- $P(A)$ is called a probability function if it satisfies the following axioms:

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- **Additivity:** If $A_1, A_2, \ldots$ are mutually disjoint events, then:

$$P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$$

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- If $A \subseteq B$, then $P(A) \leq P(B)$.

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- **Independence of Events:** Two events A and B are independent if:

$$P(A \cap B) = P(A) \cdot P(B).$$

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