Calculation of Skewness and Kurtosis using Pearson's Beta and Gamma Coefficients

Data Science and A.I. Lecture Series

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- It is a measure of skewness. For a symmetrical distribution, β_1 is zero.
- γ_1, the coefficient of skewness, resolves directionality:

$$\gamma_1 = \sqrt{\beta_1} = \frac{\mu_3}{(\mu_2)^{3/2}} = \frac{\mu_3}{\sigma^3}$$

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- These coefficients provide insight into the distribution shape.

Frequency Table with Derived Columns

Problem: For the given data, calculate the first four moments about the mean and find β_1, β_2, γ_1, and γ_2.
Given Data:

Marks (x)	5	10	15	20	25	30	35
Frequency (f)	4	10	20	36	16	12	2

Table: Frequency Distribution of Marks

Marks (x)	f	$d = \frac{(x-20)}{5}$	fd	fd^2	fd^3	fd^4
5	4	-3	-12	36	-108	324
10	10	-2	-20	40	-80	160
15	20	-1	-20	20	-20	20
20	36	0	0	0	0	0
25	16	1	16	16	16	16
30	12	2	24	48	96	192
35	2	3	6	18	54	162
Total	$\sum f = 100$		$\sum fd = -6$	$\sum fd^2 = 178$	$\sum fd^3 = -42$	$\sum fd^4 = 874$

Table: Frequency Distribution Table with Derived Columns:

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- The fourth moment about the mean:

$$\mu_4' = \frac{\sum fd^4}{N} \times h^4 = \frac{874}{100} \times 625 = 5462.50$$

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- The fourth moment about the mean:

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4 = 5423.5057$$

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- The standardized kurtosis coefficient:

$$\gamma_2 = \beta_2 - 3 = -0.2501 \,(PlatyKurtic)$$