Prove $-1 \leq r(X, Y) \leq 1$ for Karl Pearson's Correlation Coefficient

Data Science and A.I. Lecture Series

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Problem Statement

Prove that:

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The correlation coefficient $r(X, Y)$ is a measure of the linear relationship between two variables X and Y.

Formula for Correlation Coefficient

Step 1: Express the formula for $r(X, Y)$

- The formula for $r(X, Y)$ is:

$$r(X, Y) = \frac{\sum_{i=1}^{n}(x_i - \overline{X})(y_i - \overline{Y})}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{X})^2 \sum_{i=1}^{n}(y_i - \overline{Y})^2}}$$

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- Let:

$$x_i - \overline{X} = a_i \quad \text{and} \quad y_i - \overline{Y} = b_i$$

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$$x_i - \overline{X} = a_i \quad \text{and} \quad y_i - \overline{Y} = b_i$$

- Substituting:

$$r(X, Y) = \frac{\sum_{i=1}^{n} a_i b_i}{\sqrt{\sum_{i=1}^{n} a_i^2 \sum_{i=1}^{n} b_i^2}}$$

Proof using Cauchy-Schwarz Inequality

Step 2: Apply the Cauchy-Schwarz inequality

- By the Cauchy-Schwarz inequality:

$$\left(\sum_{i=1}^{n} a_i b_i\right)^2 \leq \left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} b_i^2\right)$$

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- From this, we get:

$$\left(\frac{\sum_{i=1}^{n} a_i b_i}{\sqrt{\sum_{i=1}^{n} a_i^2 \sum_{i=1}^{n} b_i^2}}\right)^2 \leq 1$$

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$$(r(X,Y))^2 \leq 1$$

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- Thus:

$$(r(X, Y))^2 \leq 1$$

Step 3: Conclude the proof

$$-1 \leq r(X, Y) \leq 1$$

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