Covariance Explained: Change of Origin vs. Scale Made Simple!

Data Science and A.I. Lecture Series

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Covariance Independence Theorem

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- Where a, b, h, k are constants.

Change of Origin Transformation

- Substituting $x_i = a + hu_i$:

$$x_i = a + hu_i - - - (1) \quad \Rightarrow \frac{1}{n}\sum x_i = a + h\frac{1}{n}\sum u_i \quad \Rightarrow \overline{X} = a + h\overline{U} - - - (2)$$

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- From the substitutions: We get

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- Covariance formula becomes:

$$\mathrm{Cov}(X, Y) = \frac{1}{n}\sum_{i=1}^{n} h(u_i - \overline{U}) \cdot k(v_i - \overline{V})$$

Simplification of Covariance Formula

- Extracting h and k:

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- Conclusion: Covariance is independent of origin (a, b) but depends on scale (h, k).

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- Proof complete.

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