Relation Between Moments About Mean and Arbitrary Point

Data Science and A.I. Lecture Series

Bindeshwar Singh Kushwaha

PostNetwork Academy

Reach PostNetwork Academy

Website

PostNetwork Academy | www.postnetwork.co

YouTube Channel

www.youtube.com/@postnetworkacademy

PostNetwork Academy Facebook Page

www.facebook.com/postnetworkacademy

LinkedIn Page

www.linkedin.com/company/postnetworkacademy

Relation Between Moments About Mean and Arbitrary Point

The rth moment about the mean is given by:

Relation Between Moments About Mean and Arbitrary Point

The rth moment about the mean is given by:

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \overline{x})^r, r = 0, 1, \ldots$$

Relation Between Moments About Mean and Arbitrary Point

The rth moment about the mean is given by:

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \overline{x})^r, r = 0, 1, \ldots$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A + A - \overline{x})^r$$

Relation Between Moments About Mean and Arbitrary Point

The rth moment about the mean is given by:

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \overline{x})^r, r = 0, 1, \ldots$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A + A - \overline{x})^r$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i((x_i - A) - (\overline{x} - A))^r \tag{1}$$

Relation Between Moments About Mean and Arbitrary Point

The rth moment about the mean is given by:

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \overline{x})^r, r = 0, 1, \ldots$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A + A - \overline{x})^r$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i((x_i - A) - (\overline{x} - A))^r \tag{1}$$

Relation Between Moments About Mean and Arbitrary Point

The rth moment about the mean is given by:

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \overline{x})^r, r = 0, 1, \ldots$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A + A - \overline{x})^r$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i((x_i - A) - (\overline{x} - A))^r \qquad (1)$$

If $d_i = x_i - A$, then:

$$\frac{\sum_{i=1}^{n} f_i d_i}{N} = \frac{\sum_{i=1}^{n} f_i x_i}{N} - \frac{\sum_{i=1}^{n} f_i A}{N} = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A) = \mu_1'$$

Relation Between Moments About Mean and Arbitrary Point

The rth moment about the mean is given by:

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \overline{x})^r, r = 0, 1, \ldots$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A + A - \overline{x})^r$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i((x_i - A) - (\overline{x} - A))^r \qquad (1)$$

If $d_i = x_i - A$, then:

$$\frac{\sum_{i=1}^{n} f_i d_i}{N} = \frac{\sum_{i=1}^{n} f_i x_i}{N} - \frac{\sum_{i=1}^{n} f_i A}{N} = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A) = \mu_1'$$

$$\implies \mu_1' = \frac{\sum_{i=1}^{n} f_i x_i}{N} - \frac{\sum_{i=1}^{n} f_i A}{N}$$

Relation Between Moments About Mean and Arbitrary Point

The rth moment about the mean is given by:

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \overline{x})^r, r = 0, 1, \ldots$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A + A - \overline{x})^r$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i((x_i - A) - (\overline{x} - A))^r \qquad (1)$$

If $d_i = x_i - A$, then:

$$\frac{\sum_{i=1}^{n} f_i d_i}{N} = \frac{\sum_{i=1}^{n} f_i x_i}{N} - \frac{\sum_{i=1}^{n} f_i A}{N} = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A) = \mu_1'$$

$$\implies \mu_1' = \frac{\sum_{i=1}^{n} f_i x_i}{N} - \frac{\sum_{i=1}^{n} f_i A}{N}$$

$$\implies \mu_1' = \bar{x} - \frac{NA}{N}$$

Relation Between Moments About Mean and Arbitrary Point

The rth moment about the mean is given by:

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i-\overline{x})^r, r = 0, 1, \ldots$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A + A - \overline{x})^r$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i((x_i - A) - (\overline{x} - A))^r \qquad (1)$$

If $d_i = x_i - A$, then:

$$\frac{\sum_{i=1}^{n} f_i d_i}{N} = \frac{\sum_{i=1}^{n} f_i x_i}{N} - \frac{\sum_{i=1}^{n} f_i A}{N} = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A) = \mu_1'$$

$$\implies \mu_1' = \frac{\sum_{i=1}^{n} f_i x_i}{N} - \frac{\sum_{i=1}^{n} f_i A}{N}$$

$$\implies \mu_1' = \bar{x} - \frac{NA}{N}$$

$$\implies \mu_1' = \bar{x} - A$$

Relation Between Moments About Mean and Arbitrary Point

The rth moment about the mean is given by:

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \overline{x})^r, r = 0, 1, \ldots$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A + A - \overline{x})^r$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i((x_i - A) - (\overline{x} - A))^r \qquad (1)$$

If $d_i = x_i - A$, then:

$$\frac{\sum_{i=1}^{n} f_i d_i}{N} = \frac{\sum_{i=1}^{n} f_i x_i}{N} - \frac{\sum_{i=1}^{n} f_i A}{N} = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A) = \mu_1'$$

$$\implies \mu_1' = \frac{\sum_{i=1}^{n} f_i x_i}{N} - \frac{\sum_{i=1}^{n} f_i A}{N}$$

$$\implies \mu_1' = \bar{x} - \frac{NA}{N}$$

$$\implies \mu_1' = \bar{x} - A$$

$$\implies d_i = x_i - A \text{ and } \mu_1' = \bar{x} - A$$

Relation Between Moments About Mean and Arbitrary Point

The rth moment about the mean is given by:

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \overline{x})^r, r = 0, 1, \ldots$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A + A - \overline{x})^r$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i((x_i - A) - (\overline{x} - A))^r \quad (1)$$

If $d_i = x_i - A$, then:

$$\frac{\sum_{i=1}^{n} f_i d_i}{N} = \frac{\sum_{i=1}^{n} f_i x_i}{N} - \frac{\sum_{i=1}^{n} f_i A}{N} = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A) = \mu_1'$$

$$\implies \mu_1' = \frac{\sum_{i=1}^{n} f_i x_i}{N} - \frac{\sum_{i=1}^{n} f_i A}{N}$$

$$\implies \mu_1' = \bar{x} - \frac{NA}{N}$$

$$\implies \mu_1' = \bar{x} - A$$

$\implies d_i = x_i - A$ and $\mu_1' = \bar{x} - A$

Plug values of $x_i - A = d_i$ and $\bar{x} - A = \mu_1'$ in equation(1) we will get

Relation Between Moments About Mean and Arbitrary Point

The rth moment about the mean is given by:

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \overline{x})^r, r = 0, 1, \ldots$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A + A - \overline{x})^r$$

$$u_r = \frac{1}{N}\sum_{i=1}^{n} f_i((x_i - A) - (\overline{x} - A))^r \qquad (1)$$

If $d_i = x_i - A$, then:

$$\frac{\sum_{i=1}^{n} f_i d_i}{N} = \frac{\sum_{i=1}^{n} f_i x_i}{N} - \frac{\sum_{i=1}^{n} f_i A}{N} = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - A) = \mu_1'$$

$$\implies \mu_1' = \frac{\sum_{i=1}^{n} f_i x_i}{N} - \frac{\sum_{i=1}^{n} f_i A}{N}$$

$$\implies \mu_1' = \bar{x} - \frac{NA}{N}$$

$$\implies \mu_1' = \bar{x} - A$$

$\implies d_i = x_i - A$ and $\mu_1' = \bar{x} - A$

Plug values of $x_i - A = d_i$ and $\bar{x} - A = \mu_1'$ in equation(1) we will get

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i(d_i - \mu_1')^r$$

Relation Between Moments About Mean and Arbitrary Point

We have,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i(d_i - \mu_1')^r$$

Relation Between Moments About Mean and Arbitrary Point

We have,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i(d_i - \mu_1')^r$$

Binomial theorem,

$$(a-b)^n = a^n - C_1^r a^{n-1} b^1 + C_2^r a^{n-2} b^2 + \cdots + (-1)^r b^n$$

Relation Between Moments About Mean and Arbitrary Point

We have,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i(d_i - \mu_1')^r$$

Binomial theorem,

$$(a-b)^n = a^n - C_1^r a^{n-1}b^1 + C_2^r a^{n-2}b^2 + \cdots + (-1)^r b^n$$

Using Binomial theorem,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i\left[d_i^r - C_1^r d_i^{r-1}\mu_1' + C_2^r d_i^{r-2}(\mu_1')^2 - \cdots + (-1)^r(\mu_1')^r\right]$$

where C_k^r are the coefficients in the expansion.

Relation Between Moments About Mean and Arbitrary Point

We have,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i(d_i - \mu_1')^r$$

Binomial theorem,

$$(a-b)^n = a^n - C_1^r a^{n-1} b^1 + C_2^r a^{n-2} b^2 + \cdots + (-1)^r b^n$$

Using Binomial theorem,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i \left[d_i^r - C_1^r d_i^{r-1}\mu_1' + C_2^r d_i^{r-2}(\mu_1')^2 - \cdots + (-1)^r(\mu_1')^r \right]$$

where C_k^r are the coefficients in the expansion.

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i d_i^r - C_1^r \mu_1' \frac{1}{N}\sum_{i=1}^{n} f_i d_i^{r-1} + C_2^r(\mu_1')^2 \frac{1}{N}\sum_{i=1}^{n} f_i d_i^{r-2} - \cdots + (-1)^r(\mu_1')^r$$

Relation Between Moments About Mean and Arbitrary Point

We have,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i(d_i - \mu_1')^r$$

Binomial theorem,

$$(a-b)^n = a^n - C_1^r a^{n-1}b^1 + C_2^r a^{n-2}b^2 + \cdots + (-1)^r b^n$$

Using Binomial theorem,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i\left[d_i^r - C_1^r d_i^{r-1}\mu_1' + C_2^r d_i^{r-2}(\mu_1')^2 - \cdots + (-1)^r(\mu_1')^r\right]$$

where C_k^r are the coefficients in the expansion.

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i d_i^r - C_1^r\mu_1'\frac{1}{N}\sum_{i=1}^{n} f_i d_i^{r-1} + C_2^r(\mu_1')^2\frac{1}{N}\sum_{i=1}^{n} f_i d_i^{r-2} - \cdots + (-1)^r(\mu_1')^r$$

$$\mu_r = \mu_r' - C_1^r\mu_1'\mu_{r-1}' + C_2^r(\mu_1')^2\mu_{r-2}' - \cdots + (-1)^r(\mu_1')^r$$

Relation Between Moments About Mean and Arbitrary Point

We have,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i(d_i - \mu_1')^r$$

Binomial theorem,

$$(a-b)^n = a^n - C_1^r a^{n-1} b^1 + C_2^r a^{n-2} b^2 + \cdots + (-1)^r b^n$$

In particular, for r = 2:

Using Binomial theorem,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i \left[d_i^r - C_1^r d_i^{r-1}\mu_1' + C_2^r d_i^{r-2}(\mu_1')^2 - \cdots + (-1)^r(\mu_1')^r\right]$$

where C_k^r are the coefficients in the expansion.

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i d_i^r - C_1^r\mu_1' \frac{1}{N}\sum_{i=1}^{n} f_i d_i^{r-1} + C_2^r(\mu_1')^2 \frac{1}{N}\sum_{i=1}^{n} f_i d_i^{r-2} - \cdots + (-1)^r(\mu_1')^r$$

$$\mu_r = \mu_r' - C_1^r\mu_1'\mu_{r-1}' + C_2^r(\mu_1')^2\mu_{r-2}' - \cdots + (-1)^r(\mu_1')^r$$

Relation Between Moments About Mean and Arbitrary Point

We have,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i(d_i - \mu_1')^r$$

Binomial theorem,

$$(a-b)^n = a^n - C_1^r a^{n-1} b^1 + C_2^r a^{n-2} b^2 + \cdots + (-1)^r b^n$$

Using Binomial theorem,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i \left[d_i^r - C_1^r d_i^{r-1}\mu_1' + C_2^r d_i^{r-2}(\mu_1')^2 - \cdots + (-1)^r(\mu_1')^r \right]$$

where C_k^r are the coefficients in the expansion.

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i d_i^r - C_1^r\mu_1' \frac{1}{N}\sum_{i=1}^{n} f_i d_i^{r-1} + C_2^r(\mu_1')^2 \frac{1}{N}\sum_{i=1}^{n} f_i d_i^{r-2} - \cdots + (-1)^r(\mu_1')^r$$

$$\mu_r = \mu_r' - C_1^r\mu_1'\mu_{r-1}' + C_2^r(\mu_1')^2\mu_{r-2}' - \cdots + (-1)^r(\mu_1')^r$$

In particular, for r = 2:

$$\mu_2 = \mu_2' - (\mu_1')^2$$

Relation Between Moments About Mean and Arbitrary Point

We have,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i(d_i - \mu_1')^r$$

Binomial theorem,

$$(a-b)^n = a^n - C_1^r a^{n-1} b^1 + C_2^r a^{n-2} b^2 + \cdots + (-1)^r b^n$$

Using Binomial theorem,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i \left[d_i^r - C_1^r d_i^{r-1}\mu_1' + C_2^r d_i^{r-2}(\mu_1')^2 - \cdots + (-1)^r(\mu_1')^r\right]$$

where C_k^r are the coefficients in the expansion.

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i d_i^r - C_1^r \mu_1' \frac{1}{N}\sum_{i=1}^{n} f_i d_i^{r-1} + C_2^r(\mu_1')^2 \frac{1}{N}\sum_{i=1}^{n} f_i d_i^{r-2} - \cdots + (-1)^r(\mu_1')^r$$

$$\mu_r = \mu_r' - C_1^r \mu_1' \mu_{r-1}' + C_2^r(\mu_1')^2 \mu_{r-2}' - \cdots + (-1)^r(\mu_1')^r$$

In particular, for r = 2:

$$\mu_2 = \mu_2' - (\mu_1')^2$$

For r =3:

Relation Between Moments About Mean and Arbitrary Point

We have,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i(d_i - \mu_1')^r$$

Binomial theorem,

$$(a-b)^n = a^n - C_1^r a^{n-1}b^1 + C_2^r a^{n-2}b^2 + \cdots + (-1)^r b^n$$

Using Binomial theorem,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i\left[d_i^r - C_1^r d_i^{r-1}\mu_1' + C_2^r d_i^{r-2}(\mu_1')^2 - \cdots + (-1)^r(\mu_1')^r\right]$$

where C_k^r are the coefficients in the expansion.

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i d_i^r - C_1^r\mu_1'\frac{1}{N}\sum_{i=1}^{n} f_i d_i^{r-1} + C_2^r(\mu_1')^2\frac{1}{N}\sum_{i=1}^{n} f_i d_i^{r-2} - \cdots + (-1)^r(\mu_1')^r$$

$$\mu_r = \mu_r' - C_1^r\mu_1'\mu_{r-1}' + C_2^r(\mu_1')^2\mu_{r-2}' - \cdots + (-1)^r(\mu_1')^r$$

In particular, for r = 2:

$$\mu_2 = \mu_2' - (\mu_1')^2$$

For r =3:

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3$$

Relation Between Moments About Mean and Arbitrary Point

We have,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i(d_i - \mu_1')^r$$

Binomial theorem,

$$(a-b)^n = a^n - C_1^r a^{n-1} b^1 + C_2^r a^{n-2} b^2 + \cdots + (-1)^r b^n$$

Using Binomial theorem,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i \left[d_i^r - C_1^r d_i^{r-1}\mu_1' + C_2^r d_i^{r-2}(\mu_1')^2 - \cdots + (-1)^r(\mu_1')^r\right]$$

where C_k^r are the coefficients in the expansion.

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i d_i^r - C_1^r \mu_1' \frac{1}{N}\sum_{i=1}^{n} f_i d_i^{r-1} + C_2^r(\mu_1')^2 \frac{1}{N}\sum_{i=1}^{n} f_i d_i^{r-2} - \cdots + (-1)^r(\mu_1')^r$$

$$\mu_r = \mu_r' - C_1^r \mu_1' \mu_{r-1}' + C_2^r (\mu_1')^2 \mu_{r-2}' - \cdots + (-1)^r(\mu_1')^r$$

In particular, for r = 2:

$$\mu_2 = \mu_2' - (\mu_1')^2$$

For r =3:

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3$$

For r =4:

Relation Between Moments About Mean and Arbitrary Point

We have,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i(d_i - \mu_1')^r$$

Binomial theorem,

$$(a-b)^n = a^n - C_1^r a^{n-1}b^1 + C_2^r a^{n-2}b^2 + \cdots + (-1)^r b^n$$

Using Binomial theorem,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i\left[d_i^r - C_1^r d_i^{r-1}\mu_1' + C_2^r d_i^{r-2}(\mu_1')^2 - \cdots + (-1)^r(\mu_1')^r\right]$$

where C_k^r are the coefficients in the expansion.

$$\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i d_i^r - C_1^r\mu_1'\frac{1}{N}\sum_{i=1}^{n} f_i d_i^{r-1} + C_2^r(\mu_1')^2\frac{1}{N}\sum_{i=1}^{n} f_i d_i^{r-2} - \cdots + (-1)^r(\mu_1')^r$$

$$\mu_r = \mu_r' - C_1^r\mu_1'\mu_{r-1}' + C_2^r(\mu_1')^2\mu_{r-2}' - \cdots + (-1)^r(\mu_1')^r$$

In particular, for r = 2:

$$\mu_2 = \mu_2' - (\mu_1')^2$$

For r =3:

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3$$

For r =4:

$$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6(\mu_1')^2\mu_2' - 3(\mu_1')^4$$