

Question Based on Variance

Data Science and A.I. Lecture Series

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If x_i are observations and f_i are frequencies of observations then variance is

$$\text{Var}(X) = \frac{1}{N} \sum_{i=1}^n f_i (x_i - \bar{X})^2 = \frac{1}{N} (\sum_{i=1}^n f_i x_i^2) - \bar{X}^2$$

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10	8	80	1	1	
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16	5	80	7	49	
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$$\text{Var}(X) = \frac{1}{N} \sum_{i=1}^n f_i(x_i - \bar{X})^2 = \frac{754}{50} = 15.5$$