

Joint Probability Mass Function and Marginal Probability Mass Function

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If (X, Y) is a two-dimensional discrete random variable, then joint probability mass function of X and Y denoted by p_{xy} and is defined as

$$p_{xy}(x_i, y_j) = P(X = x_i, Y = y_j)$$

If you toss three coins the following sample space you will get.

$S = \{TTT, TTH, THT, THH, HTT, HTH, HHT, HHH\}$

X ---- Occurrence of heads

Y ---- Occurrence of tails

$X = \{0, 1, 2, 3\}$

$Y = \{0, 1, 2, 3\}$

Probabilities of events will be

$P(X=0)=1/8, P(X=1)=3/8, P(X=2)=3/8, P(X=3)=1/8$

$P(Y=0)=1/8, P(Y=1)=3/8, P(Y=2)=3/8, P(Y=3)=1/8$

$S = \{TTT, TTH, THT, THH, HTT, HTH, HHT, HHH\}$

And joint probabilities are

$P(X=0, Y=0)=0$
$P(X=0, Y=1)=0$
$P(X=0, Y=2)=0$
$P(X=0, Y=3)=1/8$

$P(X=1, Y=0)=0$
$P(X=1, Y=1)=0$
$P(X=1, Y=2)=3/8$
$P(X=1, Y=3)=0$

$P(X=2, Y=0)=0$
$P(X=2, Y=1)=3/8$
$P(X=2, Y=2)=0$
$P(X=2, Y=3)=0$

$P(X=3, Y=0)=1/8$
$P(X=3, Y=1)=0$
$P(X=3, Y=2)=0$
$P(X=3, Y=3)=0$

Joint Probability Function $p_{xy}(x_i, y_j)$

$Y \backslash X$	0	1	2	3	Distribution of X
0	0	0	0	1/8	1/8
1	0	0	3/8	0	3/8
2	0	3/8	0	0	3/8
3	1/8	0	0	0	1/8
Distribution of Y	1/8	3/8	3/8	1/8	1

Marginal Probability Mass Function of X

$$p_x(0) = p_{xy}(0,0) + p_{xy}(0,1) + p_{xy}(0,2) + p_{xy}(0,3) = 0+0+0+1/8 = 1/8$$

$$p_x(1) = p_{xy}(1,0) + p_{xy}(1,1) + p_{xy}(1,2) + p_{xy}(1,3) = 0+0+3/8+0 = 3/8$$

$$p_x(2) = p_{xy}(2,0) + p_{xy}(2,1) + p_{xy}(2,2) + p_{xy}(2,3) = 0+3/8+0+0 = 3/8$$

$$p_x(3) = p_{xy}(3,0) + p_{xy}(3,1) + p_{xy}(3,2) + p_{xy}(3,3) = 1/8+0+0+0 = 3/8$$

Marginal Probability Mass Function of Y

$$p_y(0) = p_{xy}(0,0) + p_{xy}(1,0) + p_{xy}(2,0) + p_{xy}(3,0) = 0+0+0+1/8 = 1/8$$

$$p_y(1) = p_{xy}(0,1) + p_{xy}(1,1) + p_{xy}(2,1) + p_{xy}(3,1) = 0+0+3/8+0 = 3/8$$

$$p_y(2) = p_{xy}(0,2) + p_{xy}(1,2) + p_{xy}(2,2) + p_{xy}(3,2) = 0+3/8+0+0 = 3/8$$

$$p_y(3) = p_{xy}(0,3) + p_{xy}(1,3) + p_{xy}(2,3) + p_{xy}(3,3) = 1/8+0+0+0 = 3/8$$