Covariance and Correlation

$$Cov(\underbrace{X, Y}_{\text{Variables}}) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n-1} \qquad i = 1, 2, 3, ---, n$$

i	X	Y	$(x_i-\bar{x})$	$(y_i-\bar{y})$	$(x_i-\bar{x})(y_i-\bar{y})$
1	1	1	-2	-2	+4
2	2	2	-1	-1	+1
3	3	3	0	0	0
4	4	4	1	1	+1
5	5	5	2	2	+4

$$Cov(X, Y) = \frac{4+1+0+1+4}{5-1} = \frac{10}{4} = 2.5$$

X → Increasing
Y → Increasing

$$\bar{x} = \frac{1+2+3+4+5}{5} = 3$$

$$\bar{y} = \frac{1+2+3+4+5}{5} = 3$$

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i	X	Y	$(x_i-\bar{x})$	$(y_i-\bar{y})$	$(x_i-\bar{x})\cdot(y_i-\bar{y})$
1	1	5	-2	+2	-4
2	2	4	-1	+1	-1
3	3	3	0	0	0
4	4	2	1	-1	-1
5	5	1	2	-2	-4

$\bar{x} = 3$

$\bar{y} = 3$

$$Cov(X, Y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

$$= \frac{-4-1+0-1-4}{5-1}$$

$$= \frac{-10}{4} = -2.5$$

X → Increases
Y → Decreases

Correlation

$$Cor(X, Y) = \frac{Cov(X, Y)}{\sigma_X \ \sigma_Y} \qquad [-1, 1]$$

i	X	Y	$(x_i - \bar{x})$	$(y_i - \bar{y})$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$
1	1	1	-2	-2	4	4
2	2	2	-1	-1	1	1
3	3	3	0	0	0	0
4	4	4	1	1	1	1
5	5	5	2	2	4	4

$$Cov(X, Y) = 2.5$$

$$\sigma_X = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} \qquad \sigma_Y = \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{n-1}}$$

$$\sigma_X = \sqrt{\frac{4+1+0+1+4}{5-1}} = \sqrt{\frac{10}{4}} = \sqrt{2.5}$$

$$\sigma_Y = \sqrt{\frac{4+1+0+1+4}{5-1}} = \sqrt{\frac{10}{4}} = \sqrt{2.5}$$

$$Cov(X, Y) = \frac{2.5}{\sqrt{2.5} \times \sqrt{2.5}} = 1$$

i	X	Y	$(x_i-\bar{x})$	$(y_i-\bar{y})$	$(x_i-\bar{x})^2$	$(y_i-\bar{y})^2$
1	1	5	−2	2	+4	4
2	2	4	−1	1	+1	1
3	3	3	0	0	0	0
4	4	2	1	−1	1	1
5	5	1	2	−2	4	4
					10	10

$\boxed{\bar{x}=3 \quad \bar{y}=3}$

$$\text{Cor}(X,Y) = \frac{\text{Cov}(X,Y)}{\sigma_x \, \sigma_y}$$

$\text{Cov}(X,Y) = -2.5$

$$\sigma_x = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n-1}} \qquad \sigma_y = \sqrt{\frac{\Sigma(y_i-\bar{y})^2}{n-1}}$$

$$\sigma_x = \sqrt{\frac{10}{5-1}} = \sqrt{2.5}$$

$$\sigma_y = \sqrt{\frac{10}{5-1}} = \sqrt{2.5}$$

$$\text{Cor}(X,Y) = \frac{\text{Cov}(X,Y)}{\sigma_x \cdot \sigma_y} = \frac{-2.5}{\sqrt{2.5}\,\sqrt{2.5}} = -1$$

Pearson's correlation coefficient -

$\underline{r_{xy}} \rightarrow [-1, +1]$